

Class 1

Introduction to Nonlinear Optics

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presentations



Outline

Linear optics

Introduction to nonlinear optics

Second order nonlinearities

Third order nonlinearities

Two-photon absorption

Conclusions

Linear optics vs Nonlinear optics

Optics is a branch of physics that describes the behavior and properties of light and the interaction of light with matter. Explains optical phenomena.

Nonlinear Optics

The branch of optics that describes optical phenomena that occur when very intense light is used

Linear optics

Maxwell equations

$$\vec{\nabla} \cdot \vec{D} = \rho$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

ρ : charge density
 J : current density

$$\vec{\nabla}_x \vec{E} = -\partial \vec{B} / \partial t$$

$$\vec{\nabla}_x \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

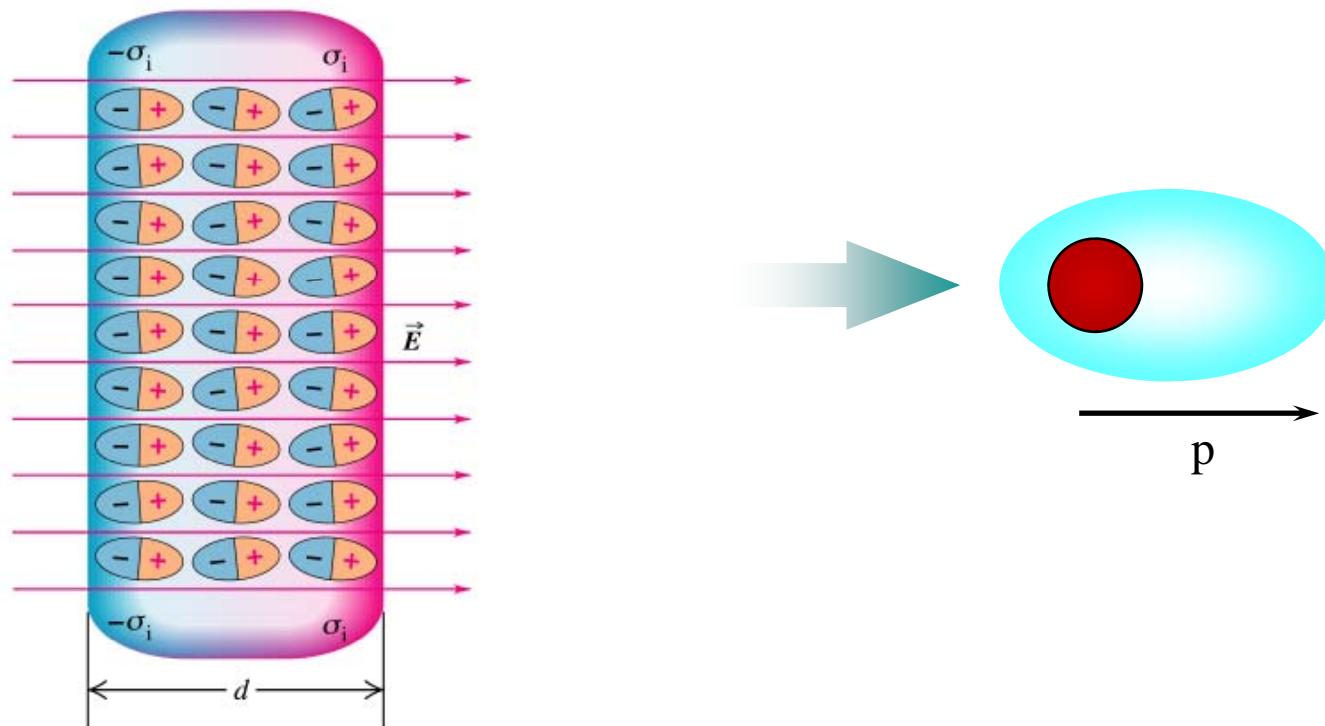
$$\vec{H} = \vec{B} / \mu_0 - \vec{M}$$

P: electric polarization
M: magnetization

Linear optics

P e M: response of the media to the applied field

Electric Polarization



Linear optics

Maxwell equations can be combined, leading to a equation
describing the electromagnetism

Constitutive relationships

$$\left. \begin{aligned} \vec{P} &= \chi \vec{E} \\ \vec{M} &= \chi_m \vec{H} \\ \vec{J} &= \sigma \vec{E} \end{aligned} \right\}$$

response of the media
to the applied field

Linear optics

wave equation ($\rho = 0; \vec{J} = 0$)

$$\nabla^2 \vec{E} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \vec{P}}{\partial t^2}$$

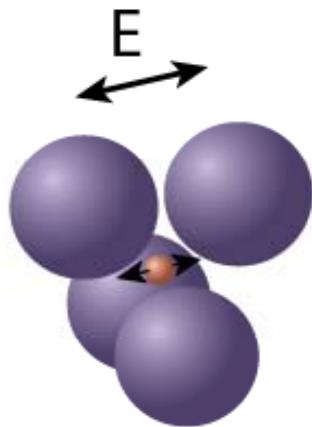
left

right

Light propagation in vacuum

Matter-light interaction

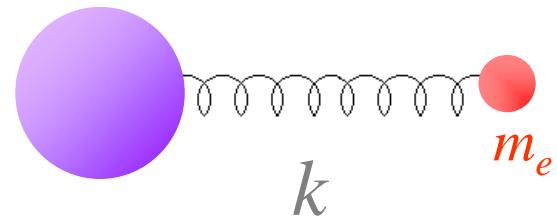
Linear optics



harmonic oscillator

$$E_{\text{rad.}} \ll E_{\text{inter.}}$$

electron on a spring



oscillation frequency

$$\omega_0 = \sqrt{\frac{k}{m_e}}$$

Linear optics

electron on a spring

$$F_{binding} = -m_e \omega_o^2 x$$

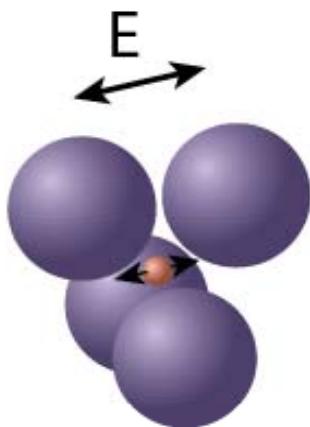
$$F_{damping} = -m_e \gamma \frac{dx}{dt}$$

$$F_{driving} = -eE = -eE_o e^{-i\omega t}$$

equation of motion

$$m \frac{d^2x}{dt^2} = \sum F$$

Linear optics



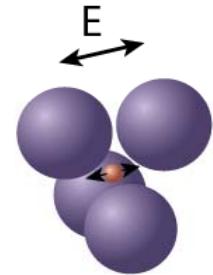
harmonic oscillator

$$m \frac{d^2x}{dt^2} + m\gamma \frac{dx}{dt} + m\omega_o^2 x = -eE$$

Steady state: electron oscillates at driving frequency

$$x(t) = x_o e^{-i\omega t} \quad x_o = -\frac{e}{m} \frac{1}{(\omega_o^2 - \omega^2) - i\gamma\omega} E_o$$

Linear optics



Oscillating dipole

$$p(t) = -e x(t) = \frac{e^2}{m} \frac{1}{(\omega_0^2 - \omega^2) - i\gamma\omega} E_o e^{-i\omega t}$$

Polarization oscillator

$$P(t) = \frac{Ne^2 / m}{(\omega_0^2 - \omega^2) - i\omega\gamma} E$$

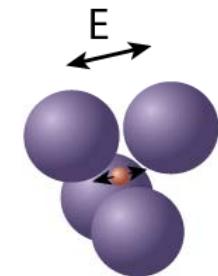
linear response

$$P = \chi E$$

Linear optics

by comparison

$$\tilde{\chi} = \frac{Ne^2 / m}{(\omega_0^2 - \omega^2) - i\omega\gamma}$$



which is a complex number

then

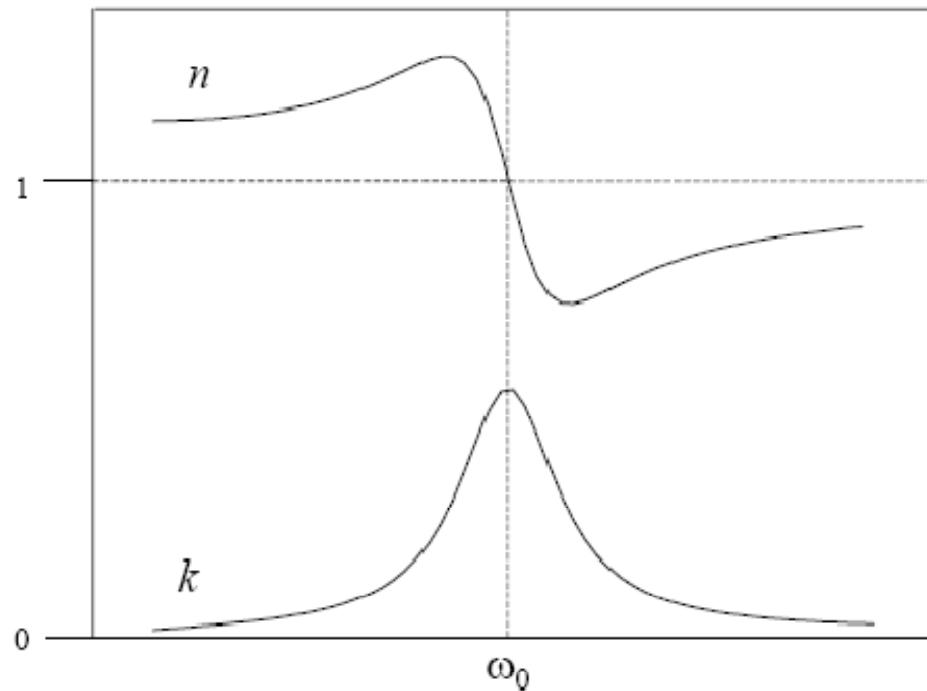
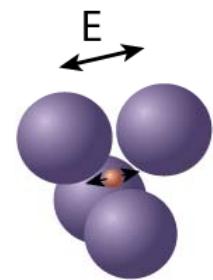
$$\tilde{n} = \sqrt{(1 + \tilde{\chi})} \approx 1 + \frac{1}{2}\tilde{\chi} + \dots = n + i\kappa$$

where n and κ are the real and imaginary parts of the complex index of refraction

refraction

absorption

Linear optics



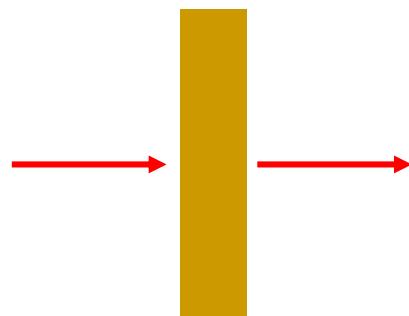
$$n = 1 + \left(\frac{Ne^2}{2m\epsilon_0} \right) \frac{(\omega_0^2 - \omega^2)}{(\omega_0^2 - \omega^2)^2 + (\omega b)^2}$$

$$\alpha(\omega) = \left(\frac{Ne^2}{m\epsilon_0} \right) \frac{\omega^2 b}{(\omega_0^2 - \omega^2)^2 + (\omega b)^2}$$

Linear optical process

absorption

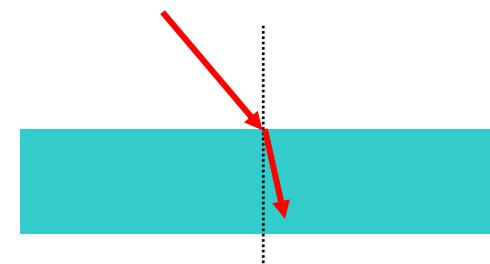
α_0 does not depend on light intensity



absorption of 10 %

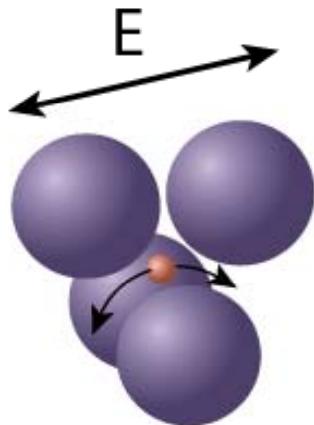
refraction

n_0 does not depend on light intensity



index of refraction 1.3

Nonlinear optics

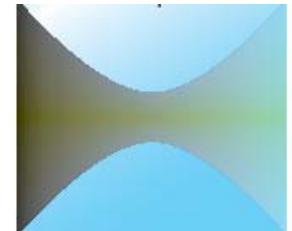


high light intensity

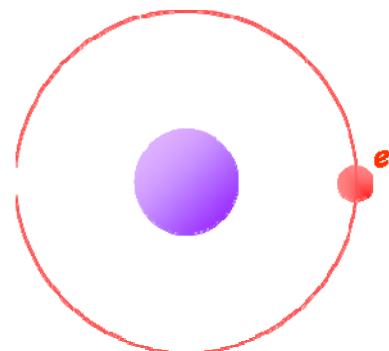
$$E_{\text{rad.}} \sim E_{\text{inter.}}$$

How high should be the light intensity ?

Nonlinear optics



Inter-atomic electric field



$$e = 1.6 \times 10^{-19} \text{ C}$$
$$r \sim 4 \text{ \AA}$$

cw laser

$$P = 20 \text{ W} \quad w_0 = 20 \text{ } \mu\text{m} \quad I = \frac{2P}{\pi w_0^2}$$

$$I = 3 \times 10^{10} \text{ W/m}^2$$

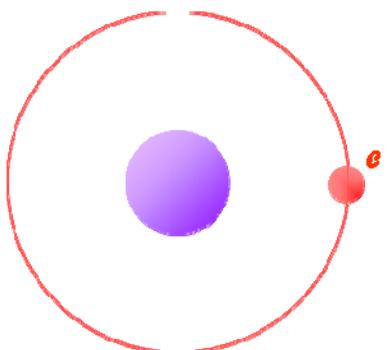
$$I = \frac{1}{2} c n \epsilon_0 E_0^2$$

$$E \sim 1 \times 10^{10} \text{ V/m}$$

$$E_0 = 4 \times 10^6 \text{ V/m}$$

Nonlinear optics

Inter-atomic electric field



$$E \sim 1 \times 10^{10} \text{ V/m}$$

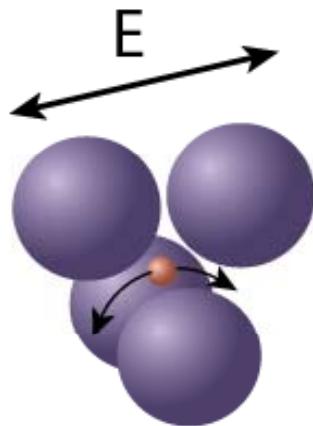
pulsed laser

$$I = 10 \text{ GW/cm}^2 = \\ 10 \times 10^{13} \text{ W/m}^2$$

$$I = \frac{1}{2} c n \epsilon_0 E_0^2$$

$$E_0 = 1 \times 10^8 \text{ V/m}$$

Nonlinear optics



high light intensity

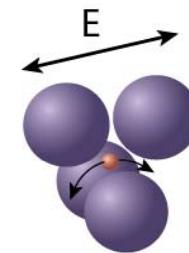
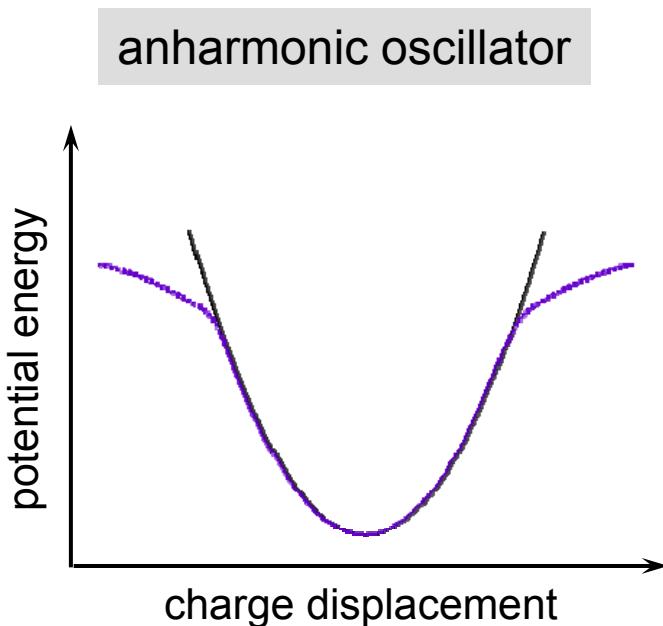
$$E_{\text{rad.}} \sim E_{\text{inter.}}$$

anharmonic oscillator

$$m \frac{d^2x}{dt^2} + m\gamma \frac{dx}{dt} + m\omega_0^2 x + \max^2 = -eE$$

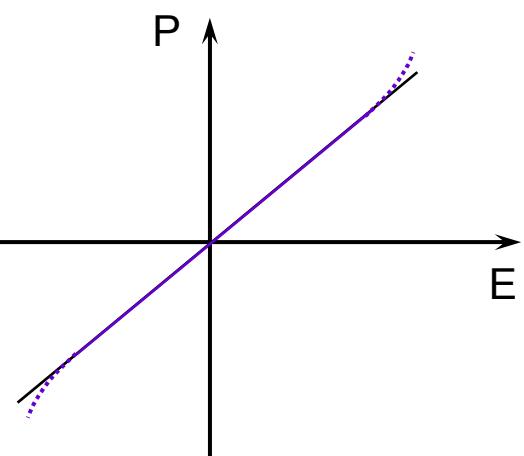
anharmonic term

Nonlinear optics

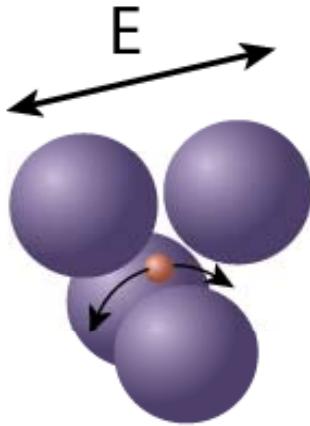


nonlinear polarization response

$$P = \chi^{(1)}E + \chi^{(2)}E^2 + \chi^{(3)}E^3 + \dots$$



Nonlinear optics



high light intensity

$$E_{\text{rad.}} \sim E_{\text{inter.}}$$

anharmonic oscillator

nonlinear polarization response

$$P = \chi^{(1)} E + \chi^{(2)} E^2 + \chi^{(3)} E^3 + \dots$$

Nonlinear optics

wave equation ($\rho = 0; \vec{J} = 0$)

$$\nabla^2 \vec{E} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \vec{P}}{\partial t^2}$$

left

right

Light propagation in vacuum

Matter-light interaction

$$\vec{P}(\vec{r}, t) = \vec{P}^{(1)}(\vec{r}, t) + \vec{P}^{NL}(\vec{r}, t)$$

Nonlinear optics

nonlinear expansion of the polarization

$$\vec{P} = \chi^{(1)} \cdot \vec{E} + \chi^{(2)} : \vec{E} \vec{E} + \chi^{(3)} : \vec{E} \vec{E} \vec{E} + \dots$$



**linear
processes**



SHG



**THG
Kerr effect**

Nonlinear optics

nonlinear expansion of the polarization

$$P = \chi^{(1)} \cdot E + \chi^{(2)} EE + \chi^{(3)} EEE +$$

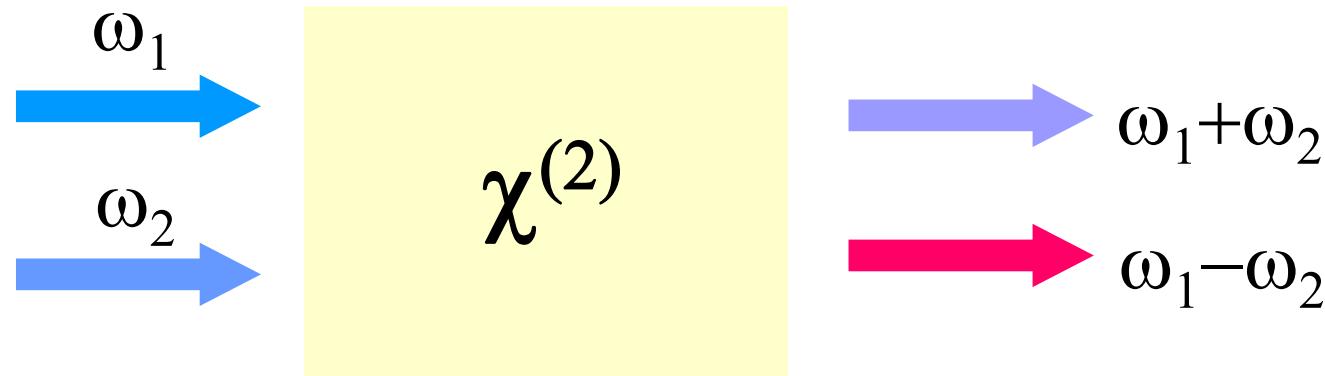
$$E_{(z,t)} = E_0 \cos(\omega t - kz)$$

$$P = \chi^{(1)} E_0 \cos(\omega t - kz) + \chi^{(2)} E_0^2 \cos^2(\omega t - kz) + \chi^{(3)} E_0^3 \cos^3(\omega t - kz) \dots$$

$$\begin{aligned} P = & \chi^{(1)} E_0 \cos(\omega t - kz) + \frac{1}{2} \chi^{(2)} E_0^2 [1 + \cos(2\omega t - 2kz)] \\ & + \chi^{(3)} E_0^3 \left[\frac{3}{4} \cos(\omega t - kz) + \frac{1}{4} \cos(3\omega t - 3kz) \right] \dots \end{aligned}$$

Nonlinear Optics

Second order processes $\chi^{(2)}$

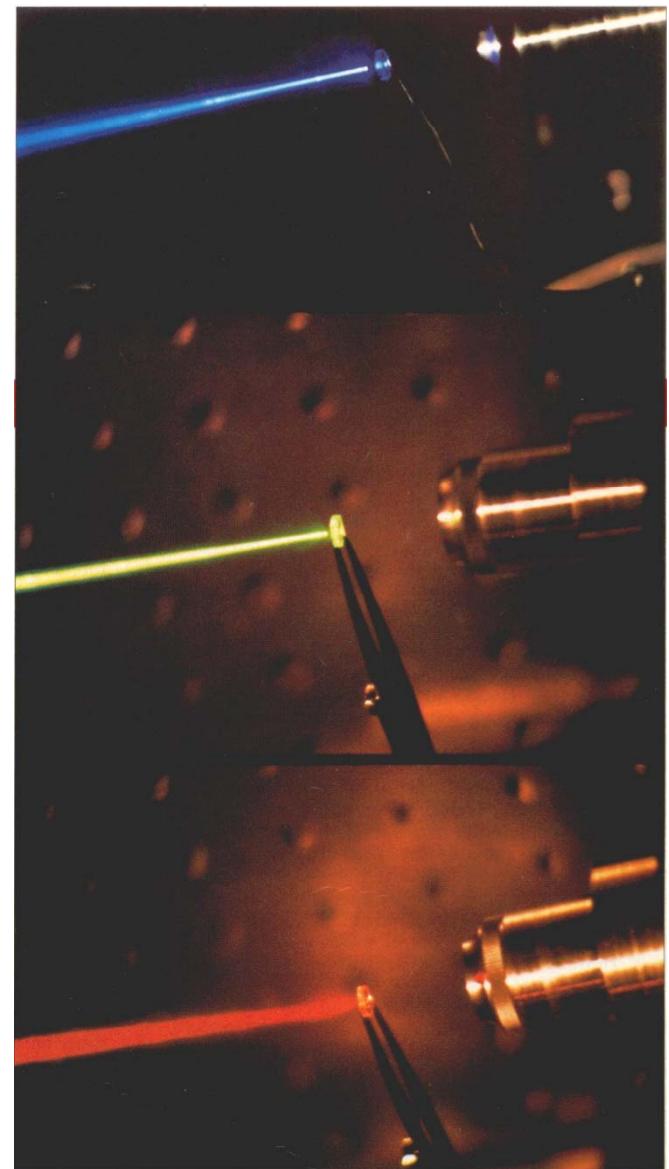
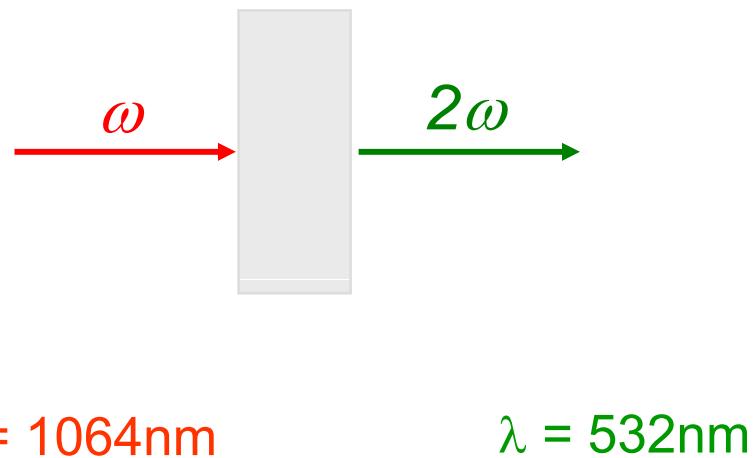


- If $\omega_1 = \omega_2$
 - second harmonic generation: 2ω
 - optical rectification: 0

Nonlinear Optics

Second order processes $\chi^{(2)}$

Second Harmonic Generation



Second Harmonic Generation

$\chi^{(2)}$



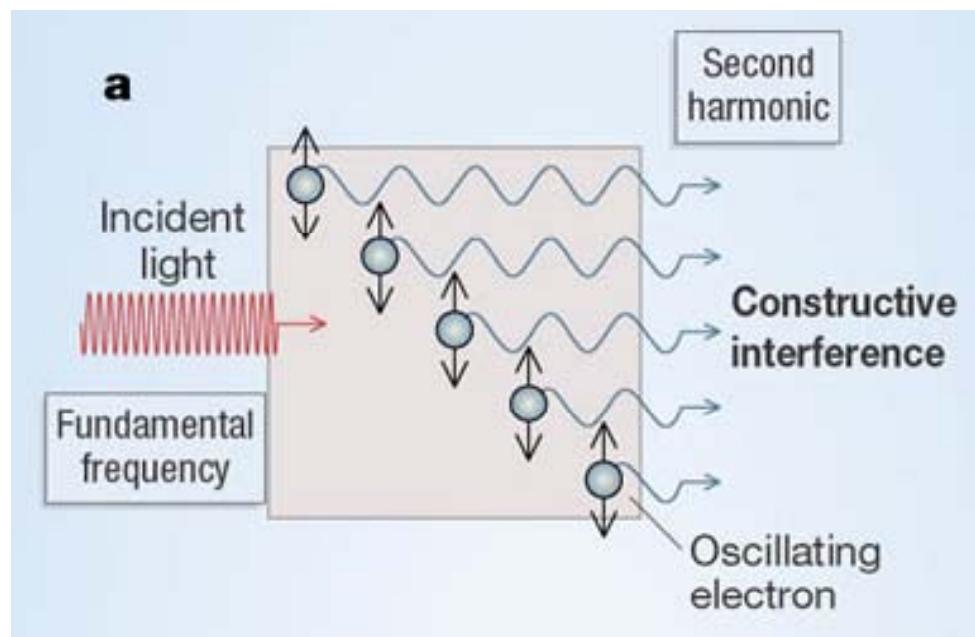
1- higher energy light

2- transparent material

Second Harmonic Generation

$\chi^{(2)}$

Phase Matching



$$v(\omega) = v(2\omega)$$

Nonlinear Optics

in medium with inversion symmetry

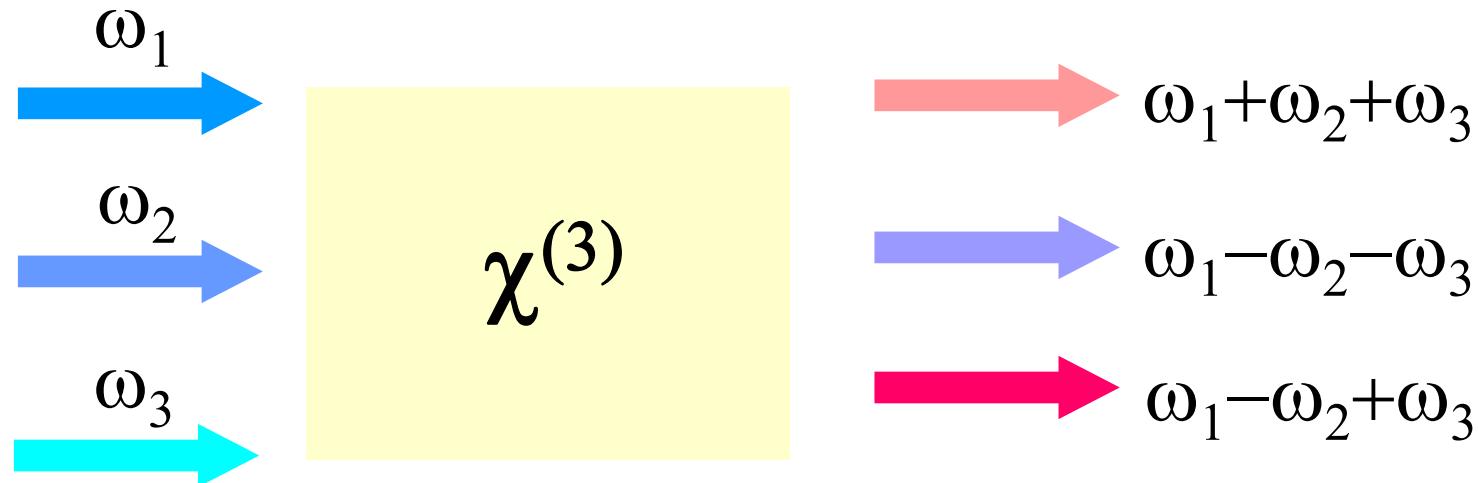
$$\vec{P}^{(2)} = \vec{\chi}^{(2)} : \vec{E} \vec{E} \Rightarrow -\vec{P}^{(2)} = \vec{\chi}^{(2)} : (-\vec{E})(-\vec{E})$$

and consequently

$$\chi^{(2)} = -\chi^{(2)} = 0$$

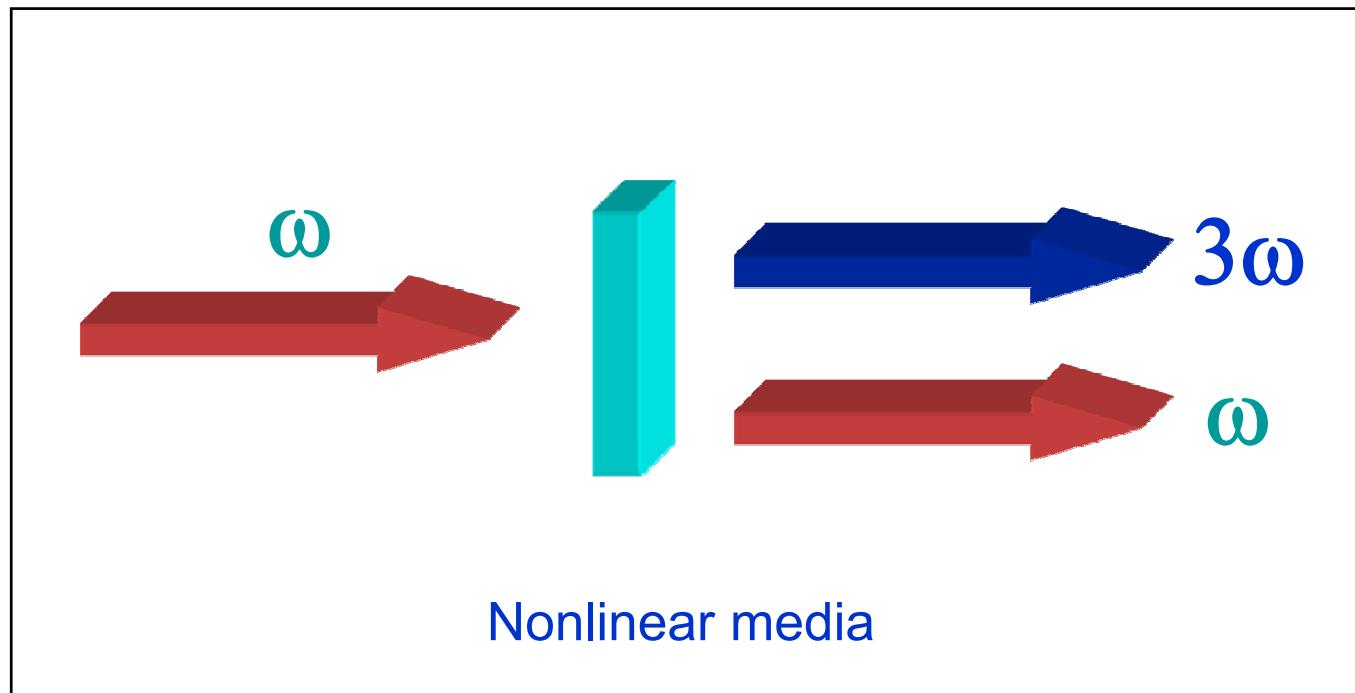
Nonlinear Optics

Third order processes $\chi^{(3)}$

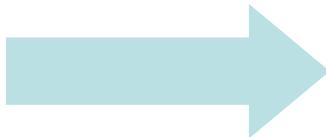


- If $\omega_1 = \omega_2 = \omega_3$
 - Third harmonic generation: 3ω
 - Self phase modulation: ω

Third Harmonic Generation $\chi^{(3)}$



Nonlinear Optics

$\chi^{(2)} = 0$  $\chi^{(3)}$ Third order processes

Nonlinear polarization

$$P = \chi^{(1)}E + \chi^{(3)}E^3 + \dots$$

Third order polarization

$$P^{(3)}(t) = \chi^{(3)}E(t)E^*(t)E(t) = \chi^{(3)}I(t)E(t)$$

Nonlinear Optics

consequently

$$P = P^{(1)} + P^{(3)} = (\chi^{(1)} + \chi^{(3)}I)E \equiv \chi_{eff}E$$

and

$$n = \sqrt{\epsilon} = \sqrt{1 + \chi_{eff}} \approx \sqrt{1 + \chi^{(1)}} + \frac{1}{2} \frac{\chi^{(3)}I}{\sqrt{1 + \chi^{(1)}}} = n_o + n_2 I$$

Kerr media

$$n = n_o + n_2 I$$

Nonlinear Optics

Third order processes $\chi^{(3)}$

$$n_2 \approx \chi^{(3)}$$

Kerr media:

$$n = n_0 + n_2 I$$

Index of refraction depends on the light intensity

Self phase modulation

Kerr media:

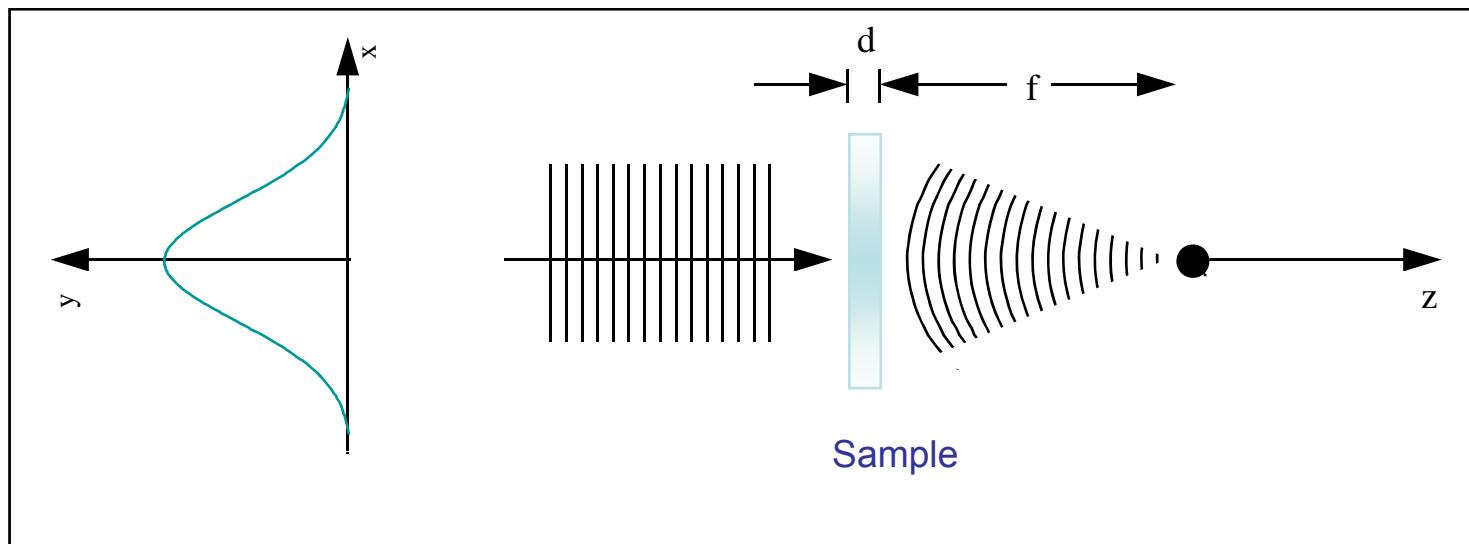
$$n = n_0 + n_2 I$$

centre symmetric: $\chi^{(2)} = 0$

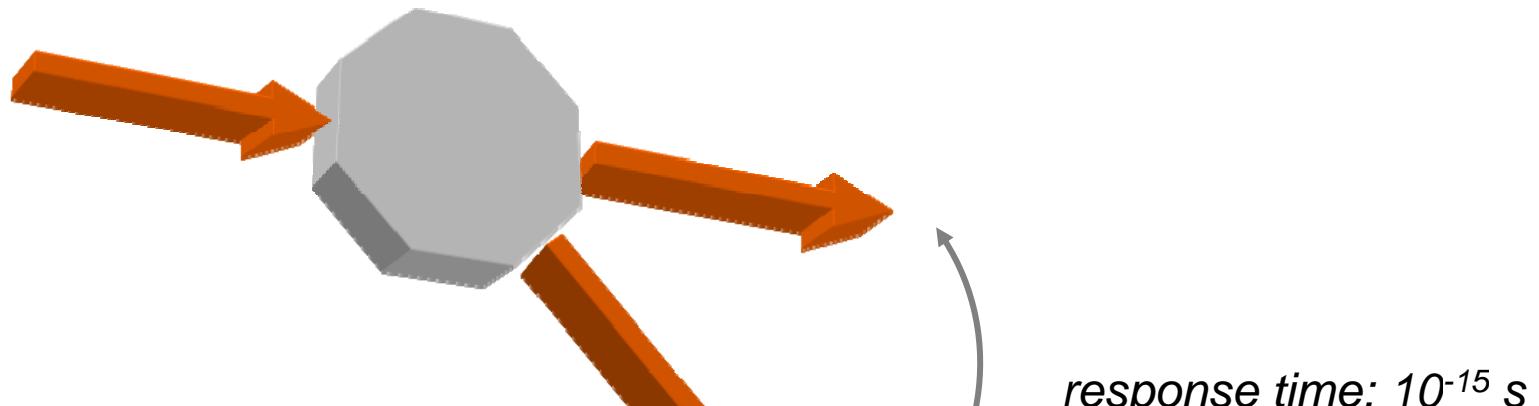
$$P_{NL} = \chi^{(3)} E^3$$

$$n_2 > 0$$

Material behaves as a convergent lens



Optical switching

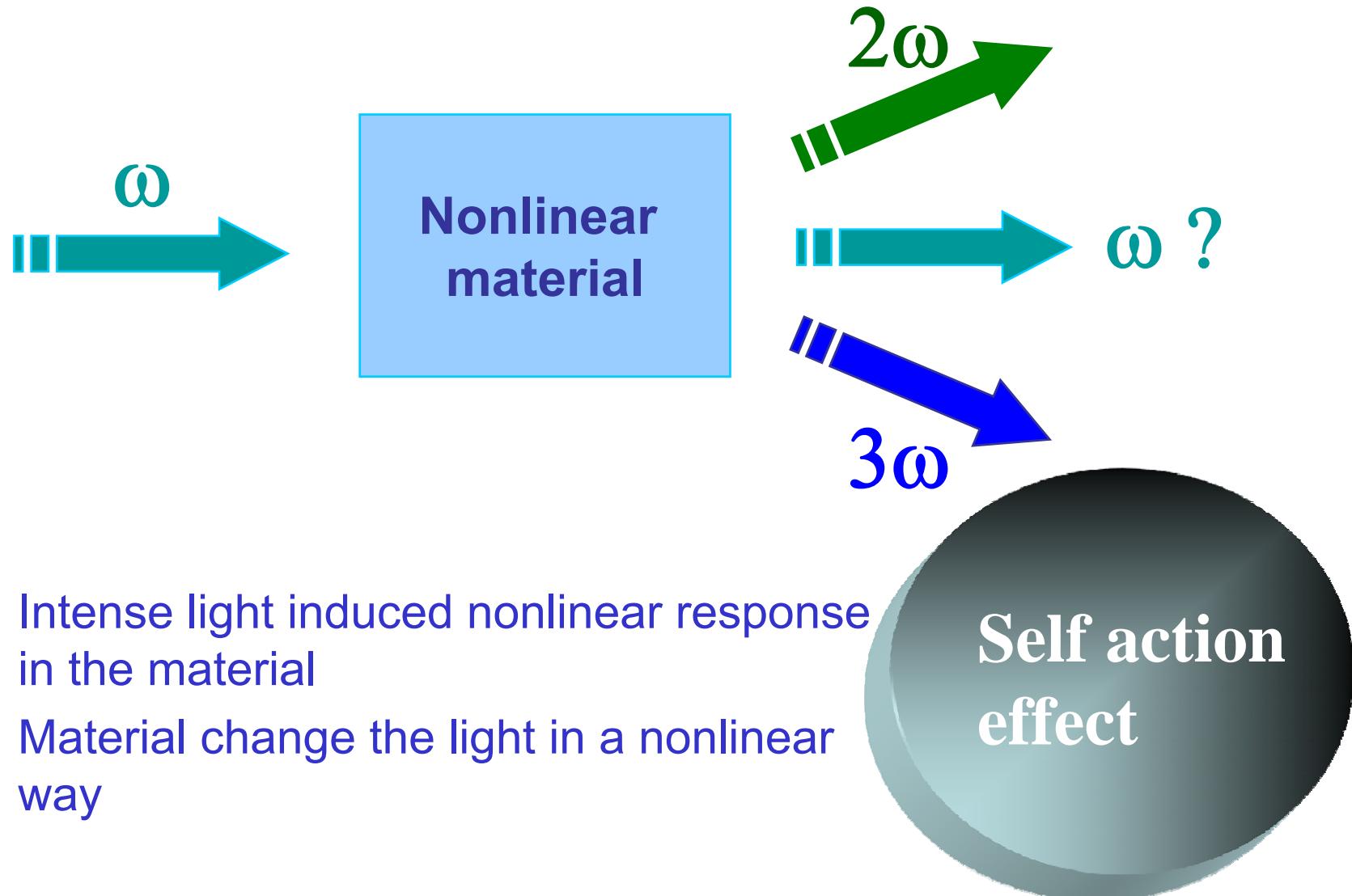


response time 1×10^{-9} 1×10^{-15}

$1\text{GHz} \rightarrow 1\text{ THz}$

1 million times faster

Nonlinear optics



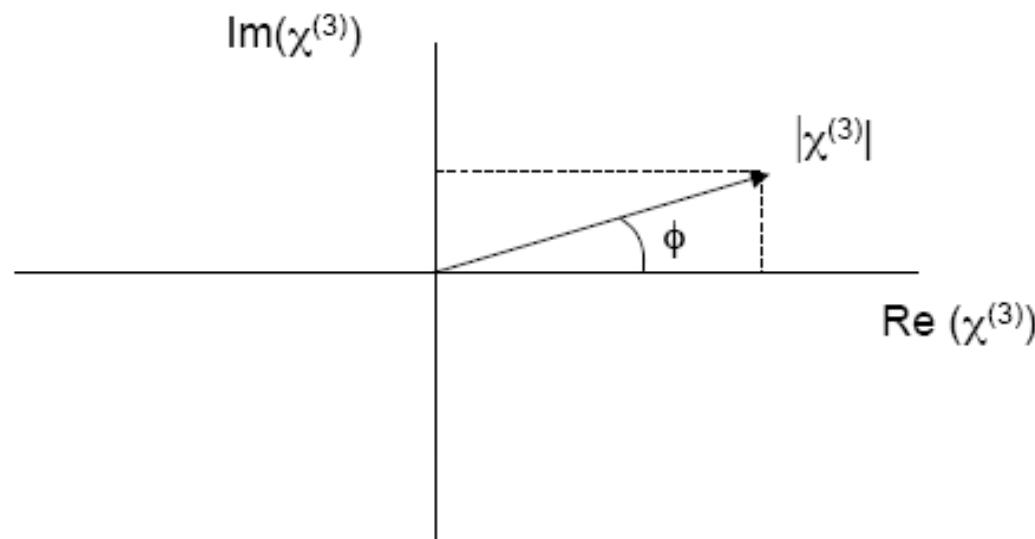
Nonlinear Optics

$\chi^{(3)}$ is a complex quantity

$$\chi^{(3)} = \text{Re}(\chi^{(3)}) + i \text{Im}(\chi^{(3)})$$

Related to intensity
dependent refractive index

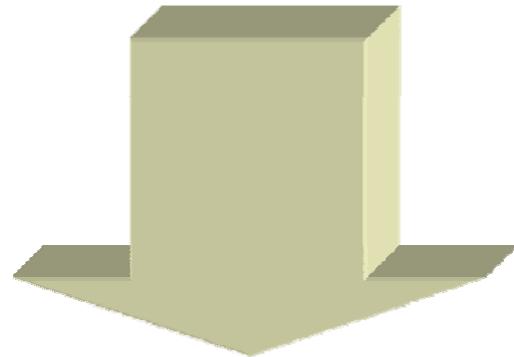
Related to two-photon
absorption



Third order processes: $\chi^{(3)}$

Refractive process:

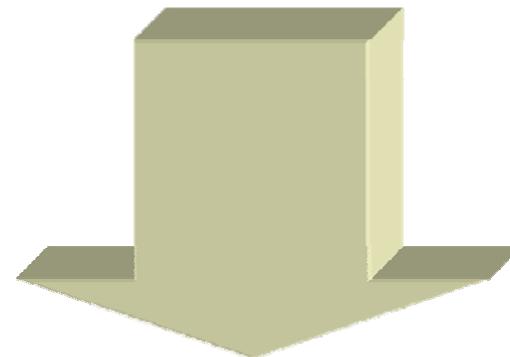
$$n=n_0+n_2 I$$



- self-phase modulation
- lens-like effect

Absorptive process:

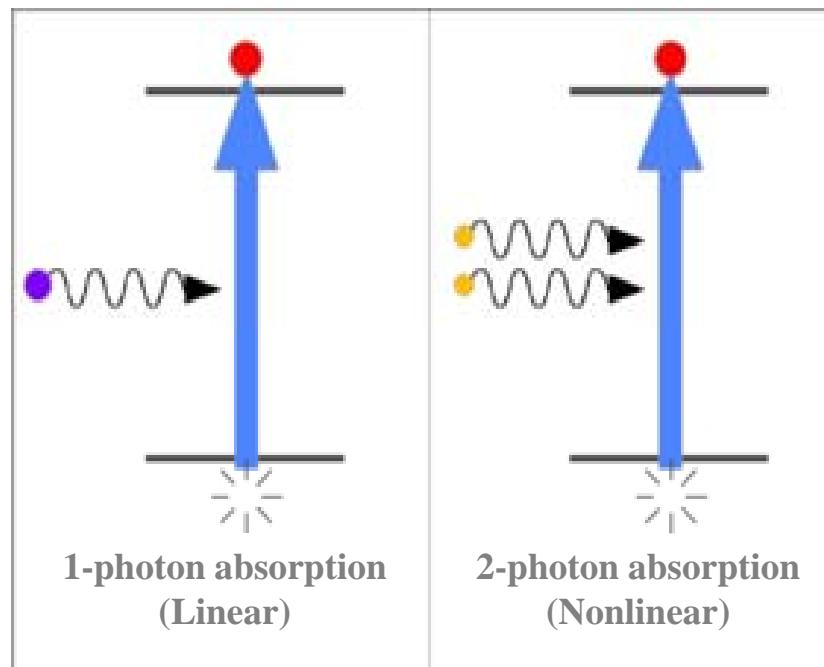
$$\alpha=\alpha_0+\beta I$$



- nonlinear absorption
- two-photon absorption

Two-photon absorption (2PA) process

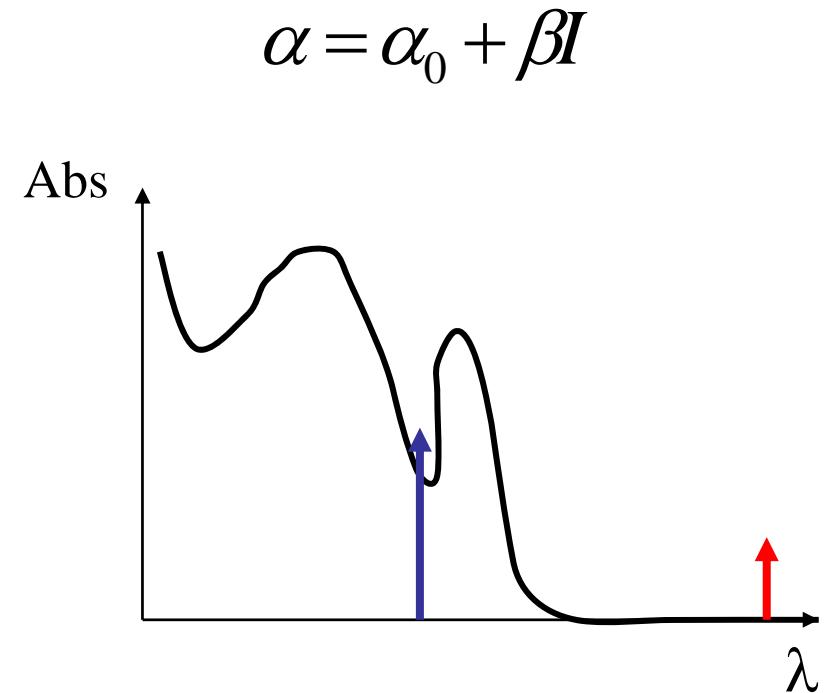
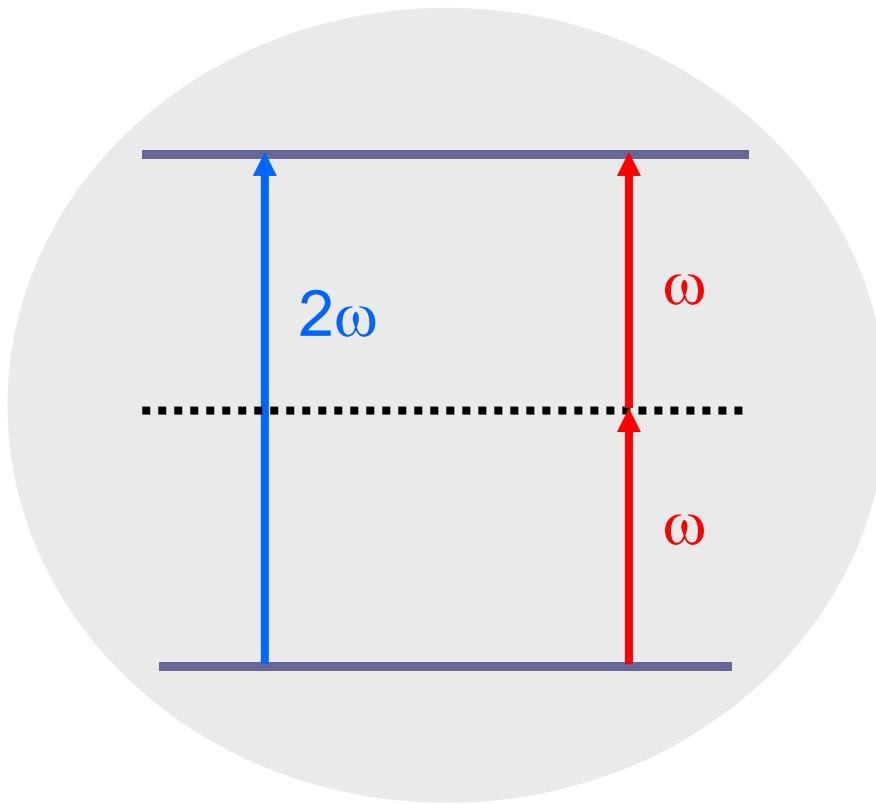
Phenomenon does not described for the Classical Physics and **does not observed until the development of the Laser.**



Theoretical model: Maria Göppert-Mayer, 1931

Two photons from an intense laser light beam are simultaneously absorbed in the same “quantum act”, leading the molecule to some excited state with energy equivalent to the absorbed two photons.

two-photon absorption



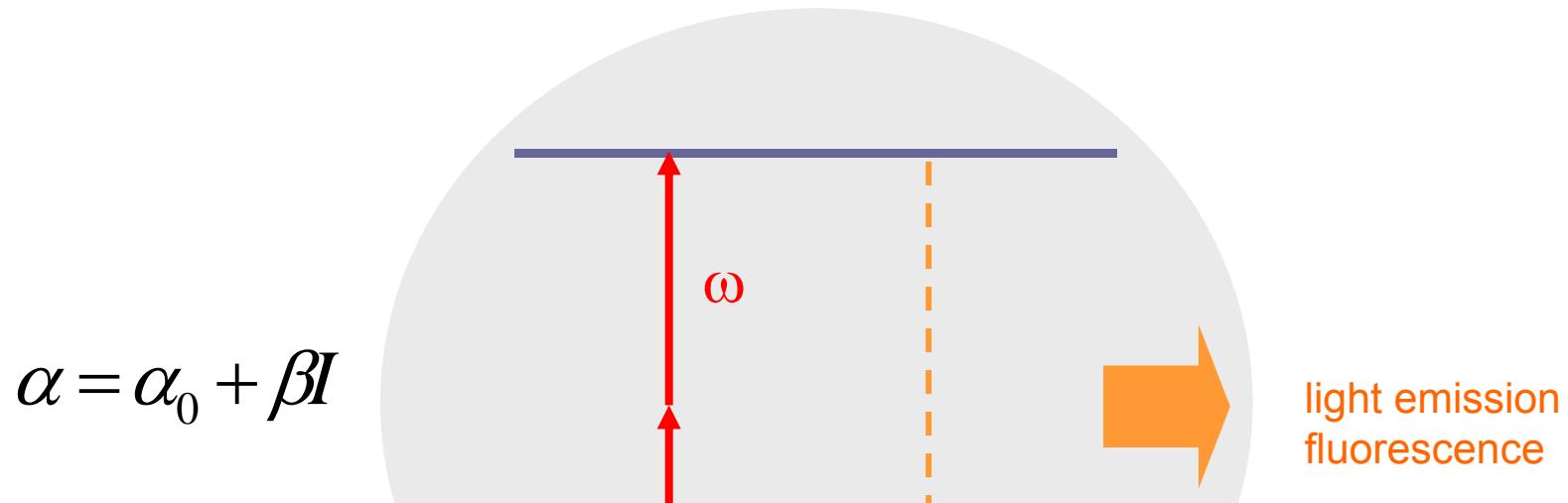
Applications:

optical limiting

fluorescence microscopy

microfabrication

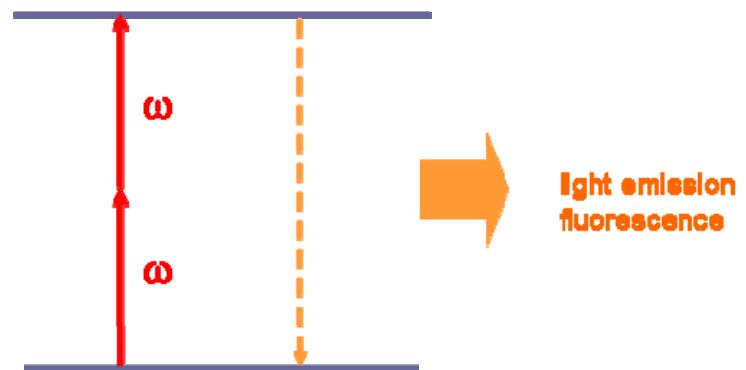
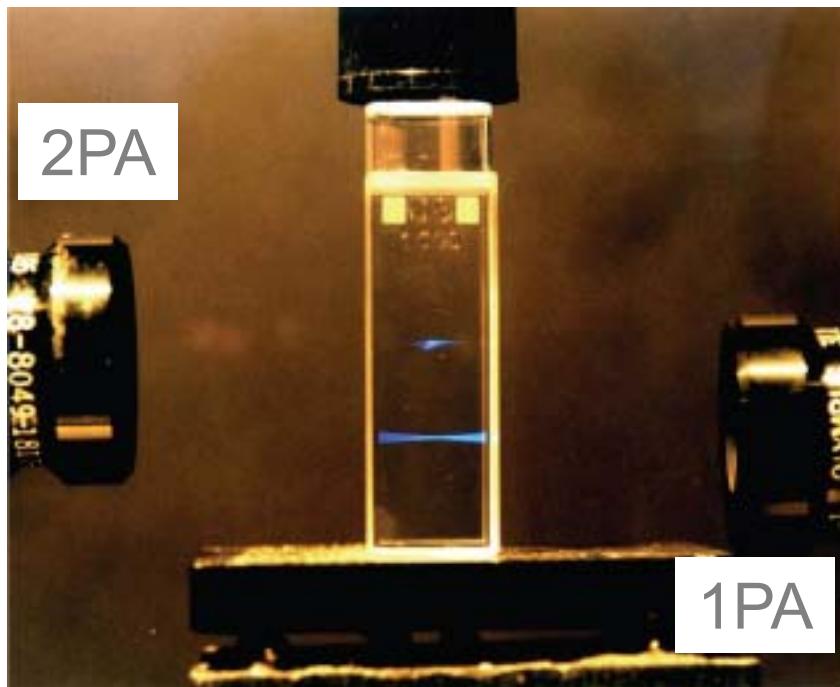
two-photon fluorescence



TPA rate constant $\propto \delta I^2$

localization of the excitation with 2PA

dilute solution of fluorescent dye



$$TPA \propto \delta I^2$$

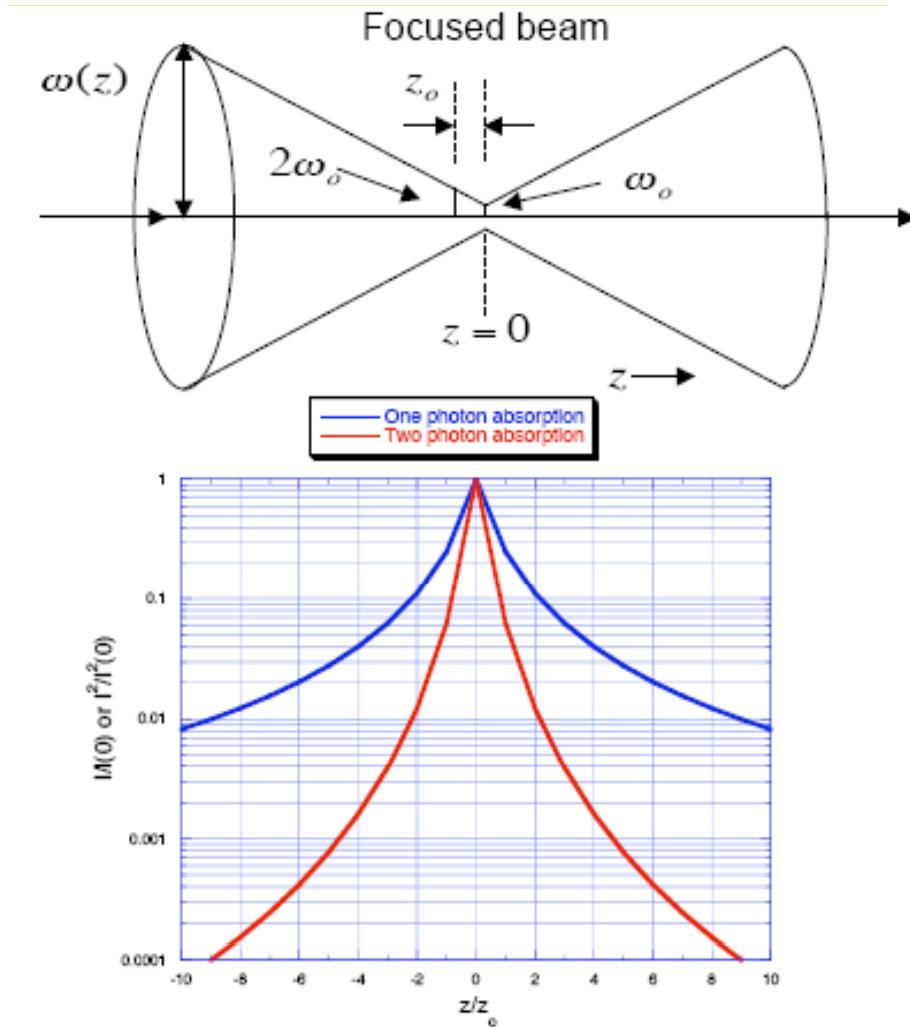
$$I \sim \frac{1}{z^2}$$

$$\Rightarrow TPA \sim \frac{1}{z^4}$$

spatial confinement of excitation

excitation profile along z

Radius, area and intensity of focused beam



$$\omega(z) = \omega_0 \left(1 + \frac{z}{z_0}\right)$$

$$A(z) = \pi \omega^2 = \pi \omega_0^2 \left(1 + \frac{z}{z_0}\right)^2$$

$$I(z) = \frac{E}{At} \propto \frac{1}{A} = \frac{1}{\pi \omega_0^2 \left(1 + \frac{z}{z_0}\right)^2}$$

Normalized excitation rates
(ignoring beam attenuation)

one photon

$$\frac{I(z)}{I(0)} = \left(1 + \frac{z}{z_0}\right)^{-2}$$

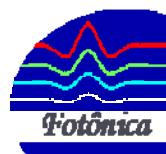
two photon

$$\frac{I(z)}{I(0)} = \left(1 + \frac{z}{z_0}\right)^{-4}$$

Class 2

Studying optical nonlinearities

Prof. Cleber R. Mendonca

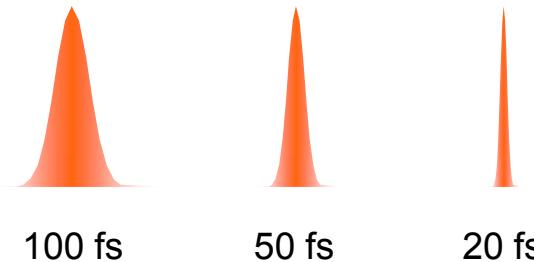


<http://www.fotonica.ifsc.usp.br>

Investigating nonlinear optical materials

Very intense light: femtosecond pulses

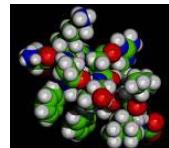
Ti:Sapphire lasers



$$1 \text{ fs} = 10^{-15} \text{ s}$$

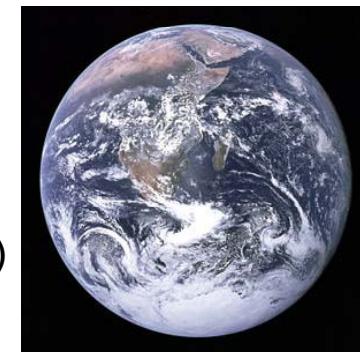
1 fs

→ 1s



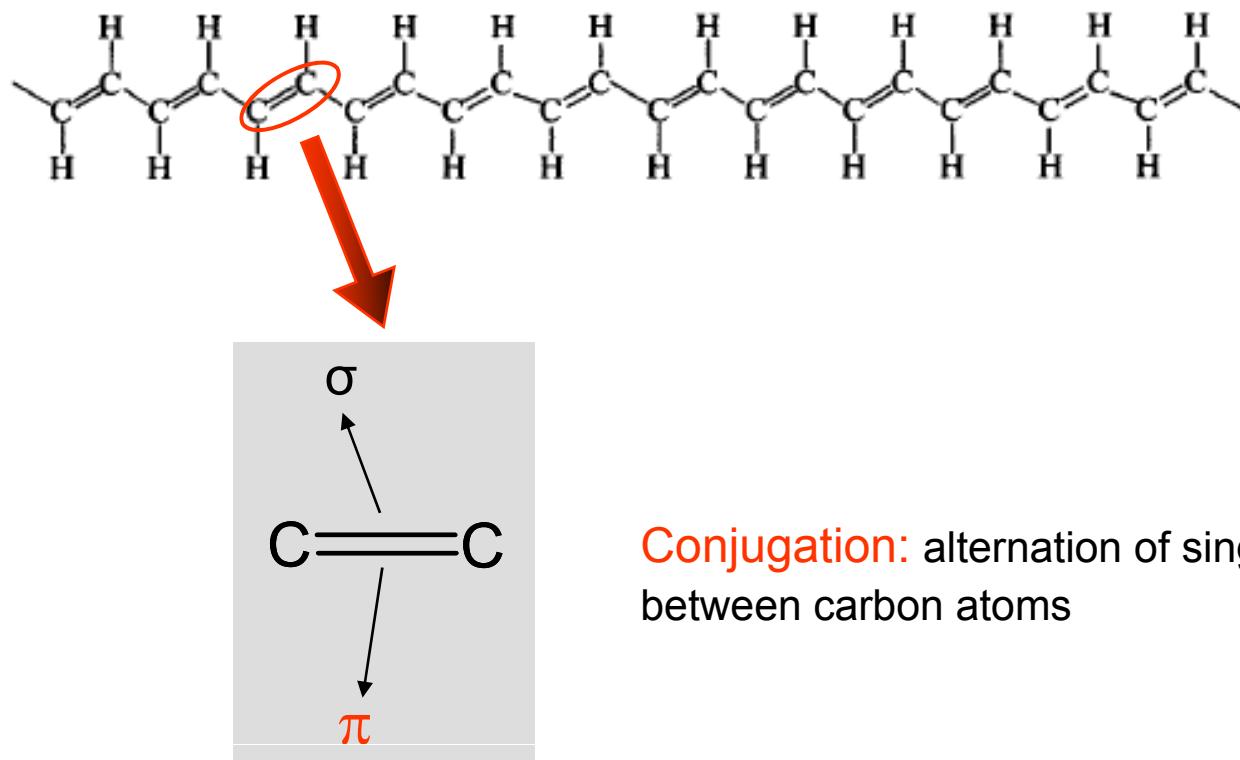
Laser intensities $\sim 100 \text{ GW/cm}^2$
 $1 \times 10^{11} \text{ W/cm}^2$

Laser pointer: 1 mW/cm^2 ($1 \times 10^{-3} \text{ W/cm}^2$)



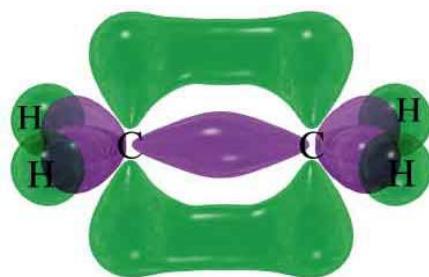
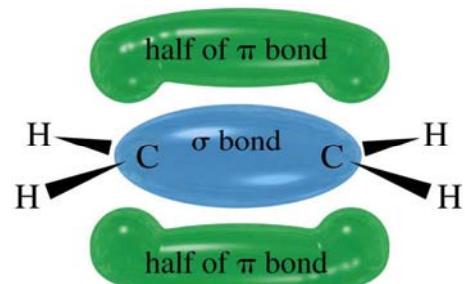
Organic materials

- Flexibility to tune the nonlinear optical response by manipulating the molecular structure
- π -conjugated structures



Conjugation: alternation of single and doubles bonds between carbon atoms

π -conjugation



σ bond: forms a strong chemical bond; localized

π bond: weaker bond; out of the C atoms axis



"Free electrons" that are easier to move under an applied electric field

π bond in conjugated system: delocalized electrons

high optical nonlinearities

A red upward-pointing arrow is positioned to the left of the equation $\chi^{(3)}$.

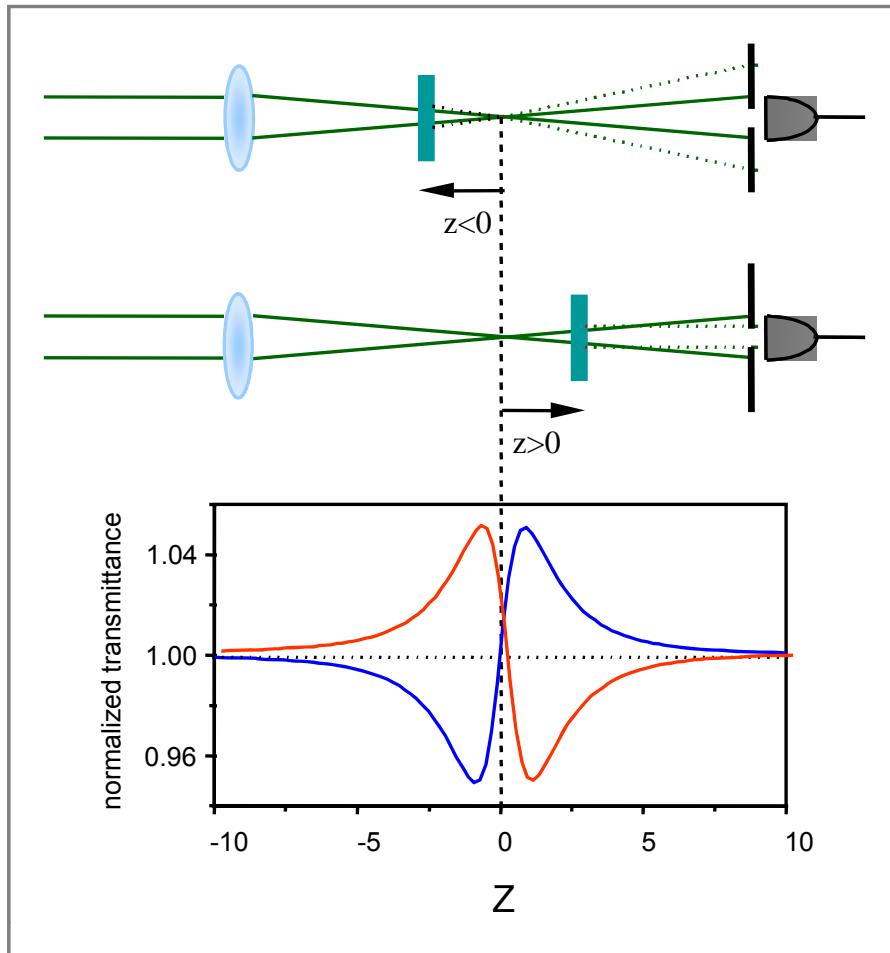
$$\chi^{(3)}$$

Research

- Understanding the physical principles behind two-photon absorption
- Understanding the relationship between molecular structure and two-photon absorption
- Developing molecules with high optical nonlinearities that can be used for application

Z-scan

closed aperture Z-scan

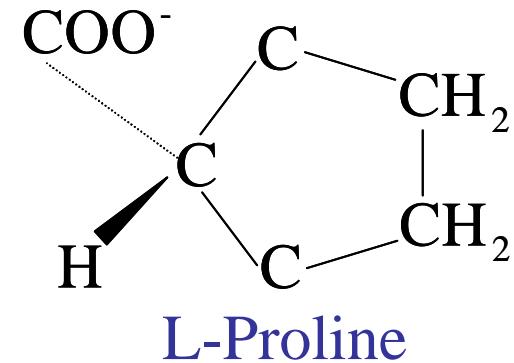
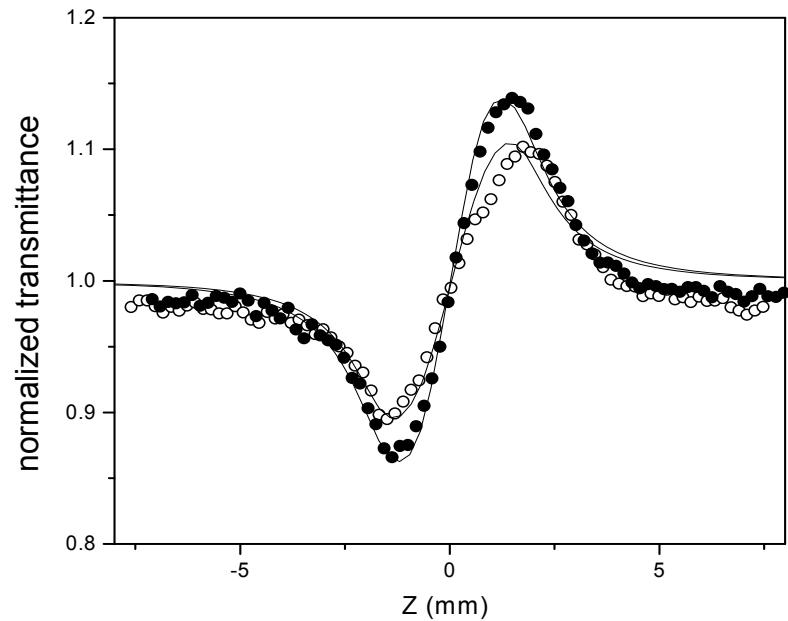


$$n = n_0 + n_2 I$$

$$\Delta T \propto n_2 I$$

Nonlinear refraction

Aminoacids



$$\Delta T_{pv} = 0.406 \Delta \Phi_0 = 0.406 \left(\frac{2\pi}{\lambda} n_2 I_0 L \right)$$

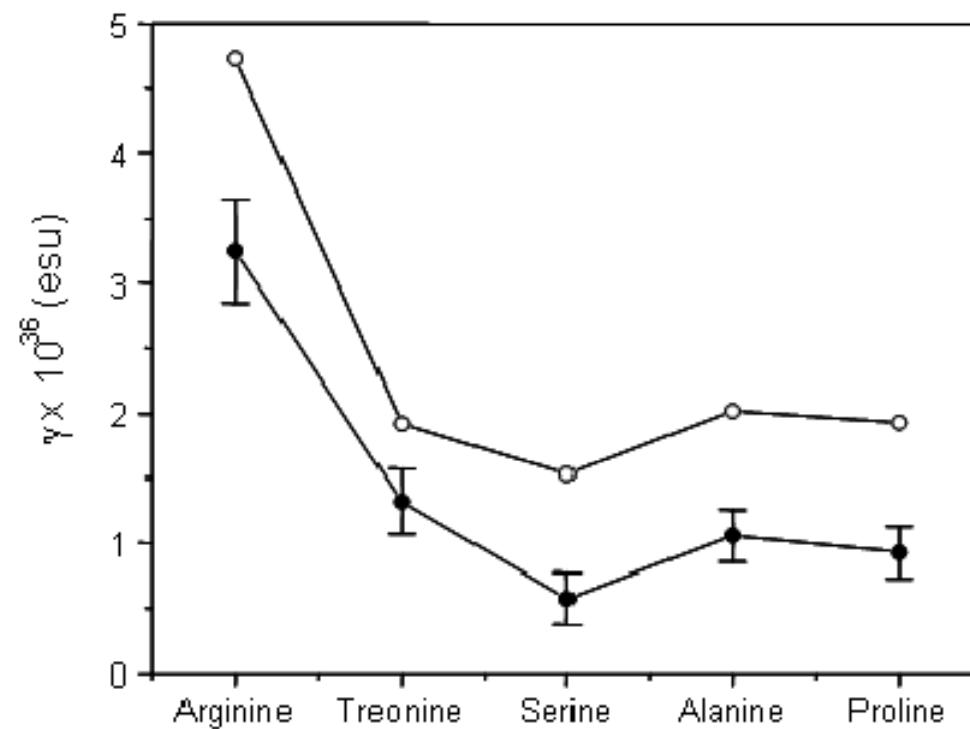
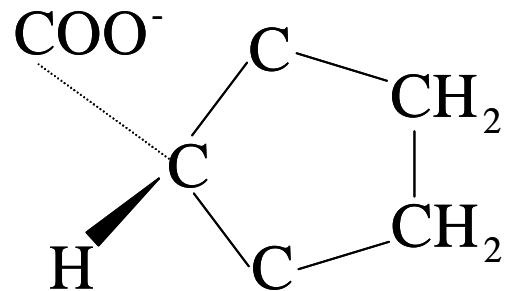
150 fs 100 GW/cm²

775 nm

$$n_2 \propto \chi^{(3)}$$

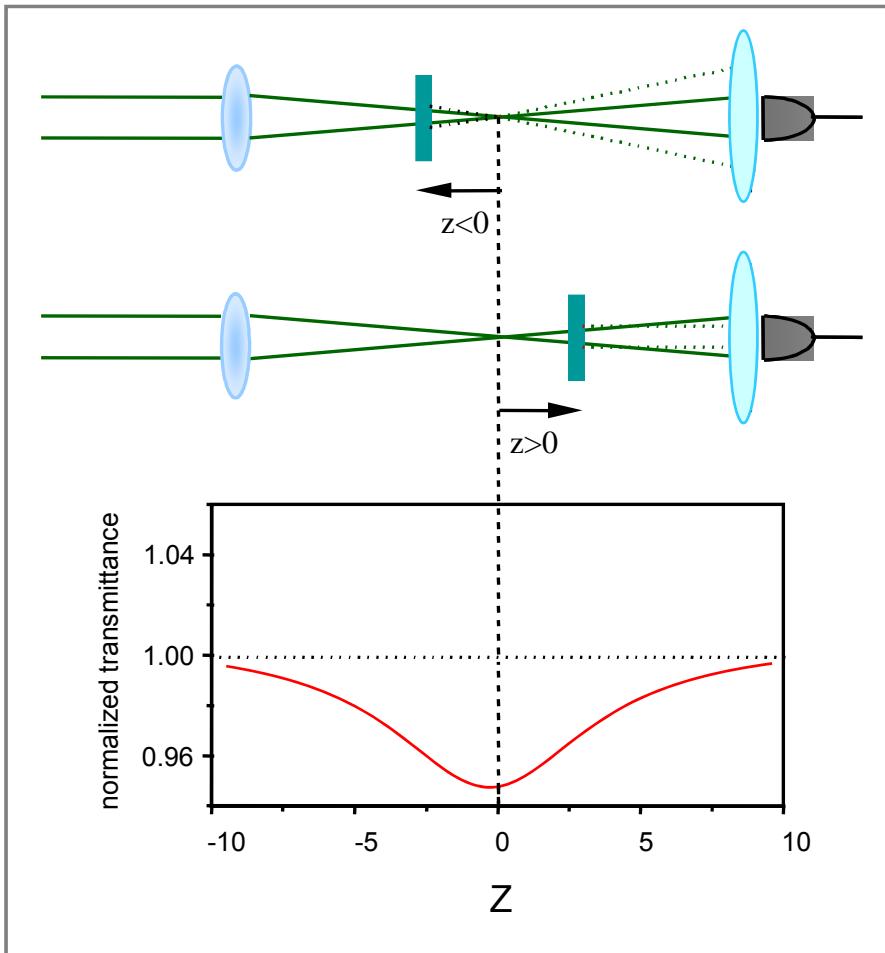
Nonlinear refraction

L-Proline



Z-scan (nonlinear absorption)

open aperture Z-scan



$$\alpha(I) = \alpha_0 + \beta I$$

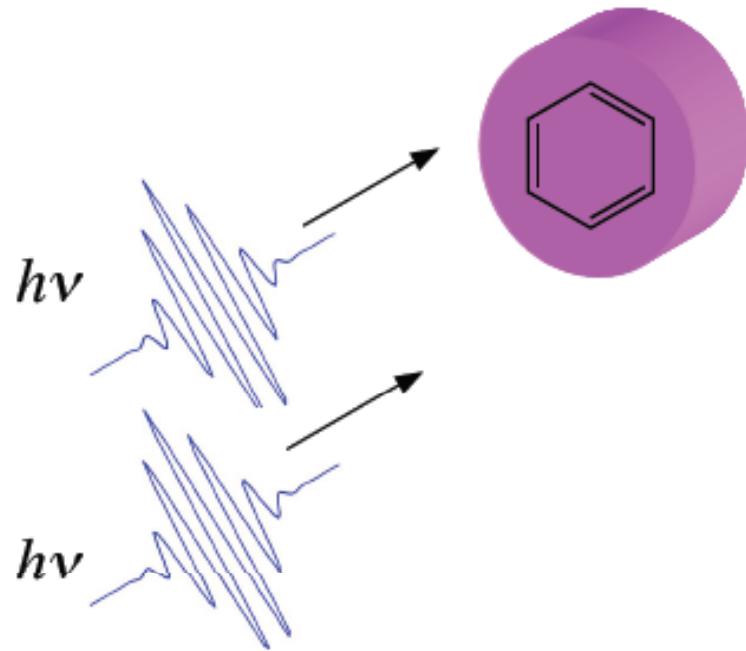
$$\Delta T \propto \beta I$$

$$T(z) = \sum_{m=0}^{\infty} \frac{[-q_0(z,0)]^m}{(m+1)^{3/2}}$$

$$q_0(z,t) = \beta I_0 L / \left(1 + z^2 / z_0^2\right)$$

two-photon absorption cross-section

A pair of photon incident on a molecule



Relation between molecular TPA cross section and TPA absorption coefficient

$$\delta = \frac{h\nu\beta}{N}$$

Representative values for TPA transitions:

$$\delta = 10 \text{ to } 1000 \times 10^{-50} \text{ cm}^4 \text{ s photon}^{-1}$$

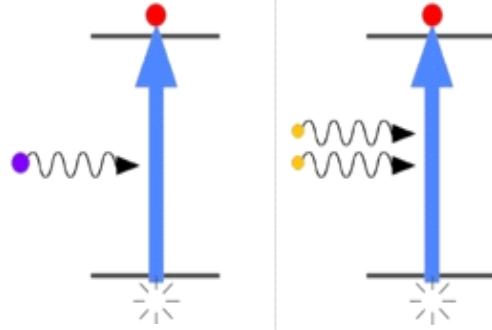
$$\beta = 10 \text{ to } 100 \text{ cm GW}^{-1} \text{ for a neat material}$$

For two-photon absorption to occur, a pair of photons must be incident within a cross sectional area and within the lifetime of the virtual state, $\tau \sim 10^{-15} \text{ s}$

Nonlinear spectrum

nonlinear absorption

$$\alpha = \alpha_0 + \beta I$$



nonlinear refraction

$$n = n_0 + n_2 I$$

intense laser (ultra short pulses)



discrete λ 's

$$\delta(\lambda) \quad n_2(\lambda)$$

nonlinear spectrum ???

Nonlinear absorption spectrum



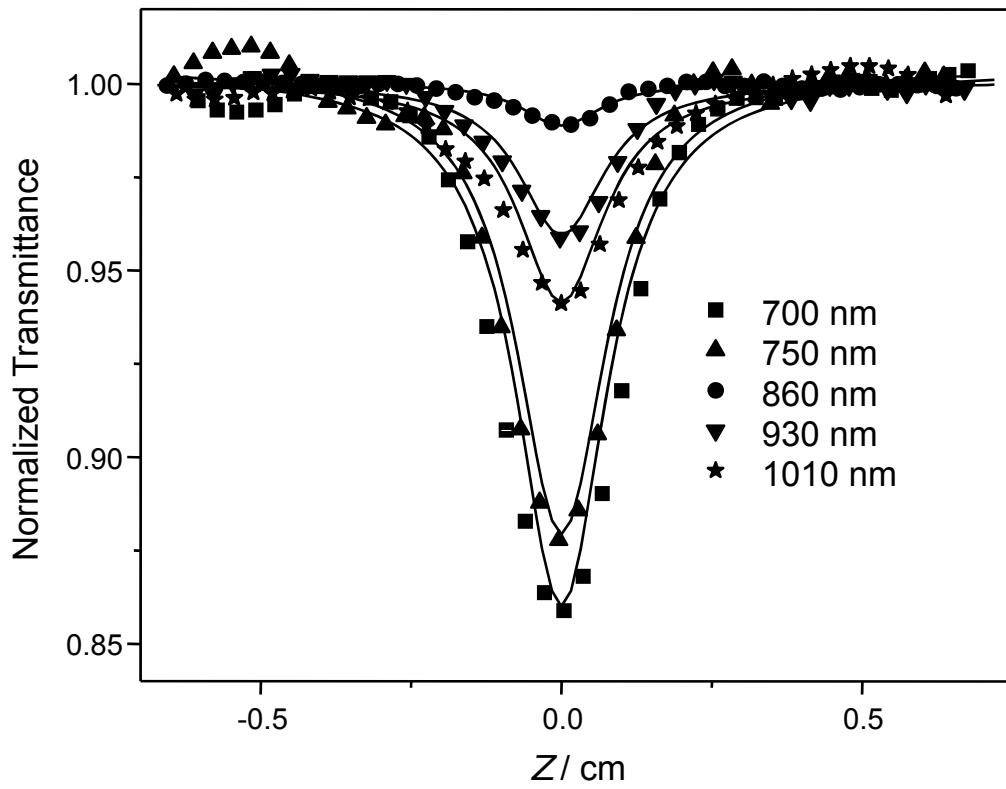
Optical parametric amplifier

$460 - 2600 \text{ nm}$

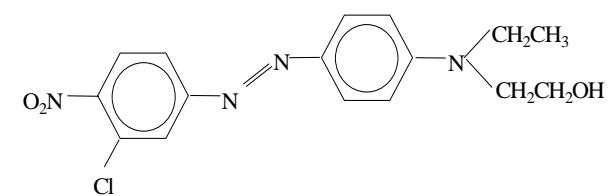
$\approx 120 \text{ fs}$

$20-60 \mu\text{J}$

Two-photon absorption



DR13

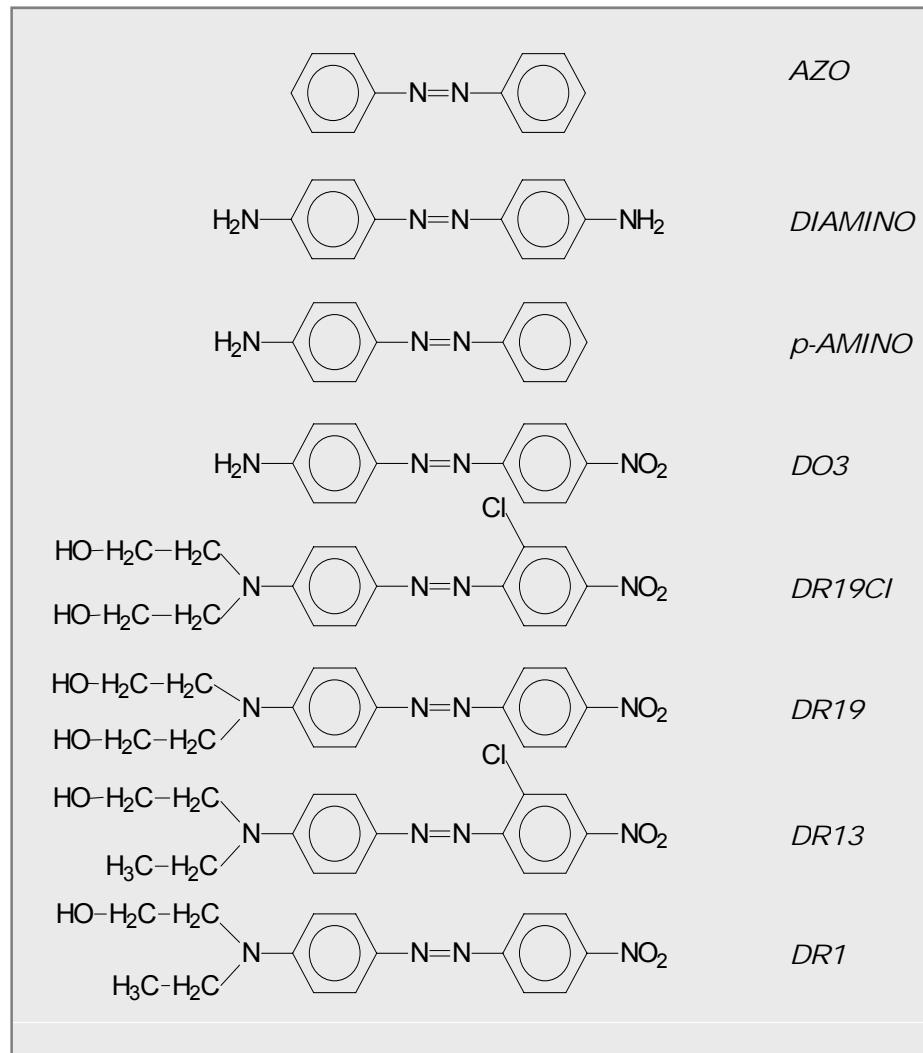


$$T(z) = \sum_{m=0}^{\infty} \frac{[-q_0(z,0)]^m}{(m+1)^{3/2}}$$

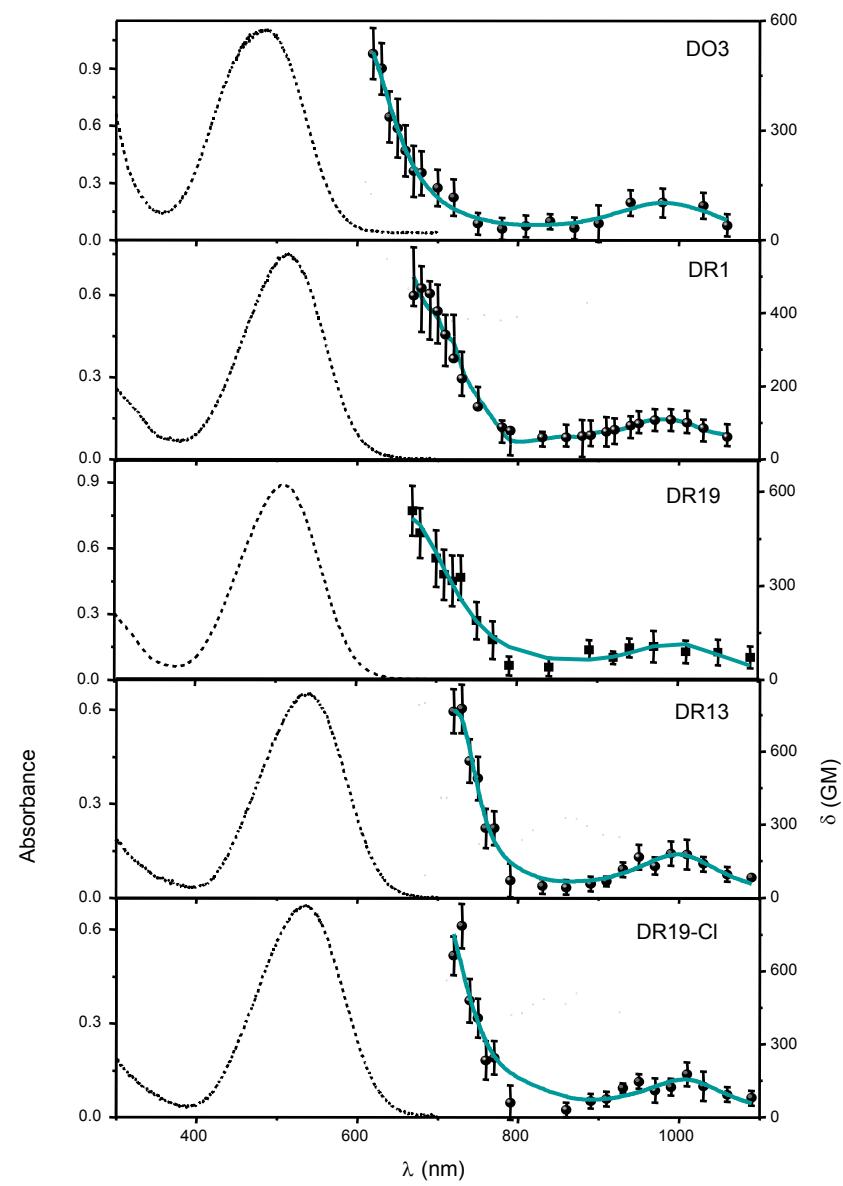
$$\alpha = \alpha_0 + \beta I$$

β : two-photon absorption coefficient

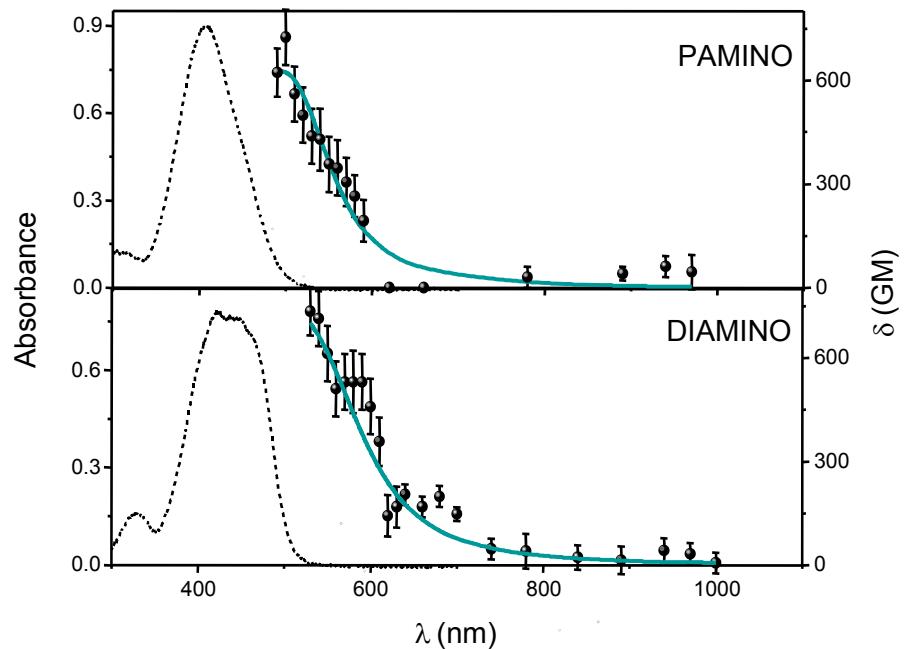
Azoaromatic samples



Psuedostilbenos

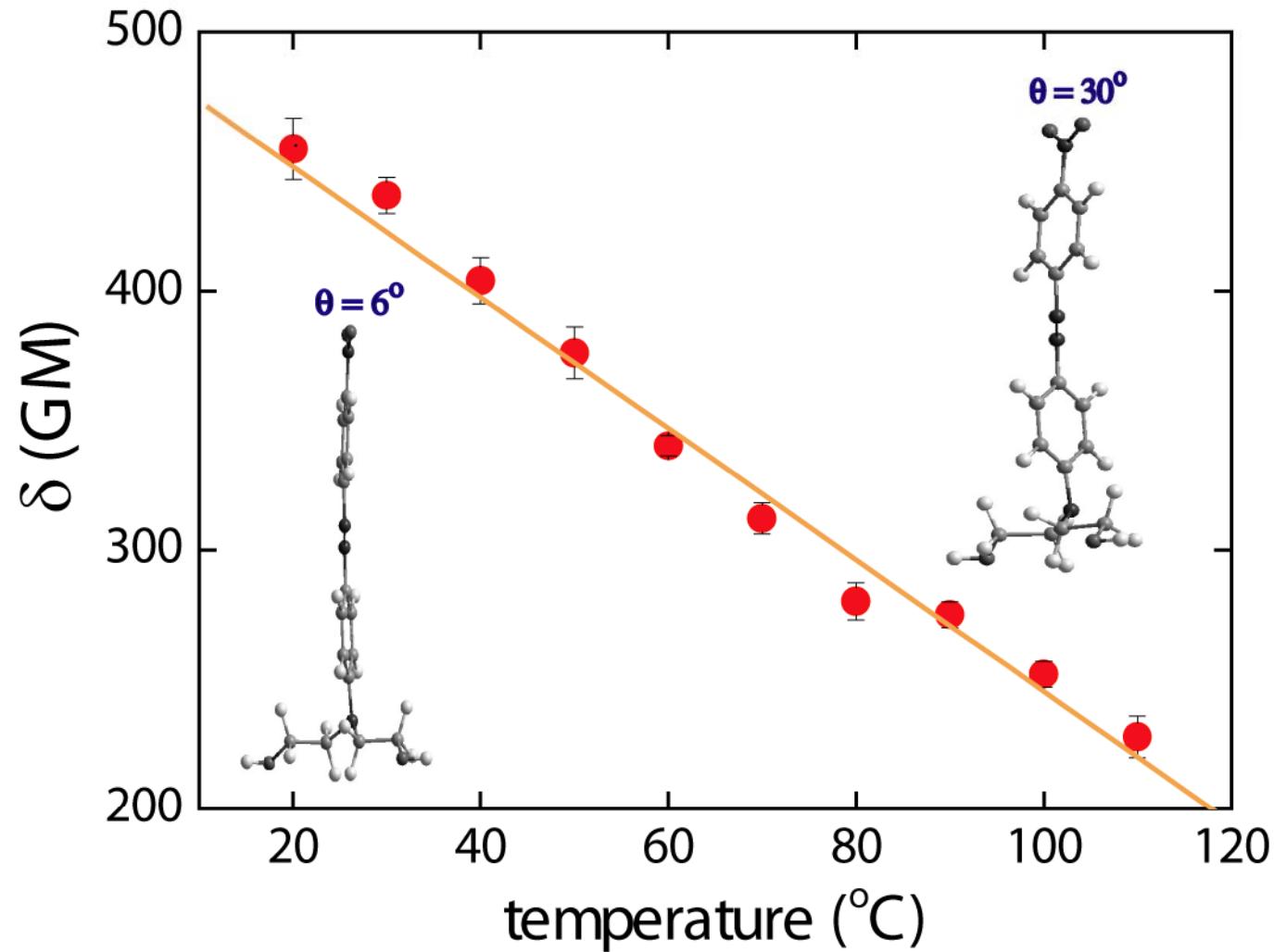


Aminoazobenzenos



$$\delta(\nu) \propto \frac{\nu^2}{(\nu_{i0} - \nu)^2 + \Gamma_{i0}^2} \left[\frac{A_1}{(\nu_{f10} - 2\nu)^2 + \Gamma_{f10}^2} + \frac{A_2}{(\nu_{f20} - 2\nu)^2 + \Gamma_{f20}^2} \right]$$

Planarity of the π -bridge

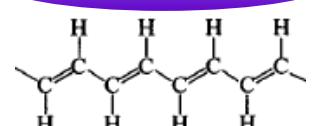
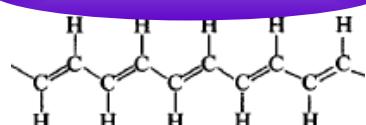
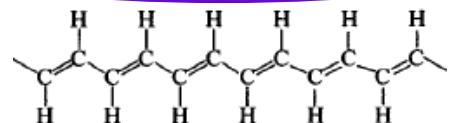
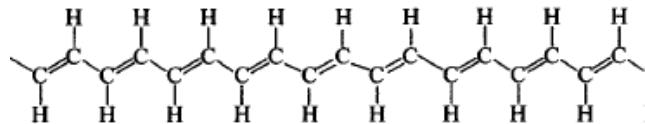


Thermally induced torsion in the molecular structure

Molecular design strategy

- Increasing the molecular conjugation
- Adding charged groups to the molecule
- Keep molecular planarity

Increasing the conjugation

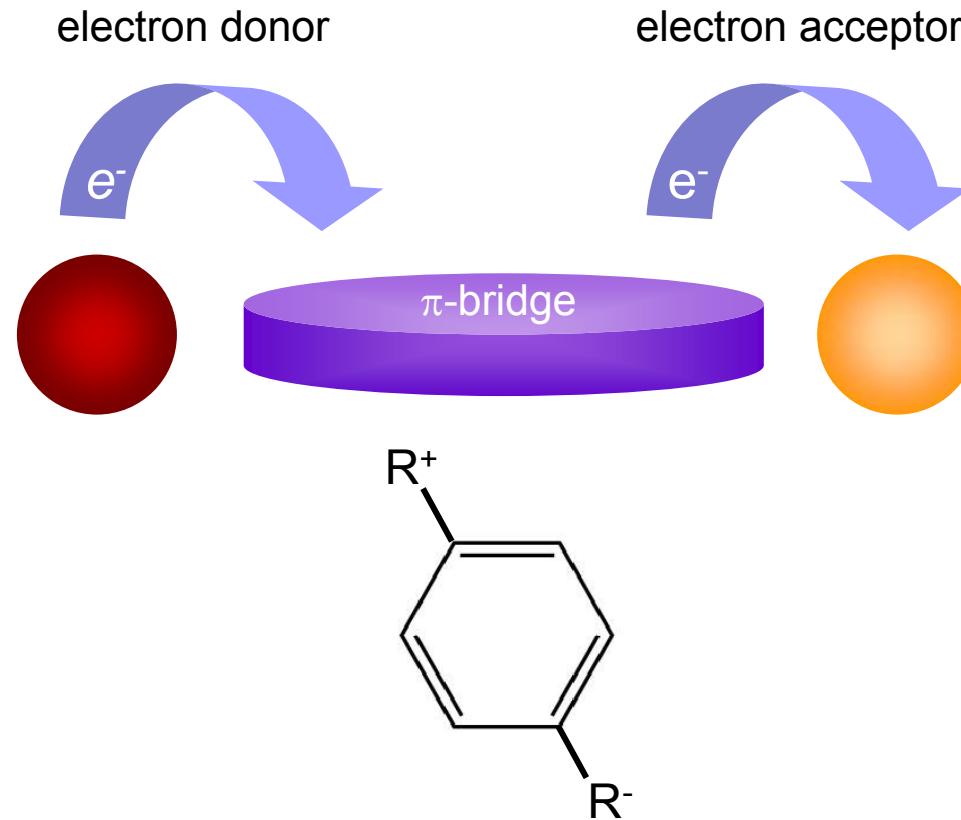


Increase in the optical nonlinearity



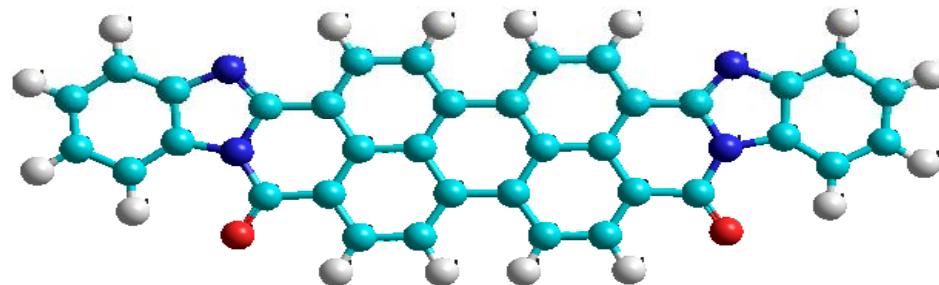
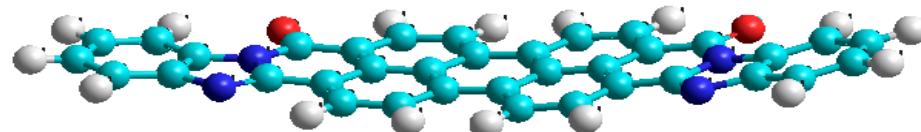
Increasing the π -conjugation

Donor and acceptor groups



Incorporating electron donor and acceptor groups in a predictable way leads to an enhancement of the optical nonlinearity

Planarity of the π -bridge



Perylene compounds are very planar molecules, which explains its high optical nonlinearities

Class 3

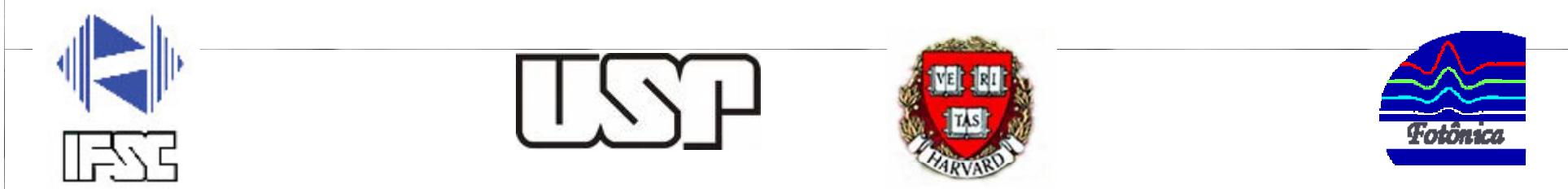
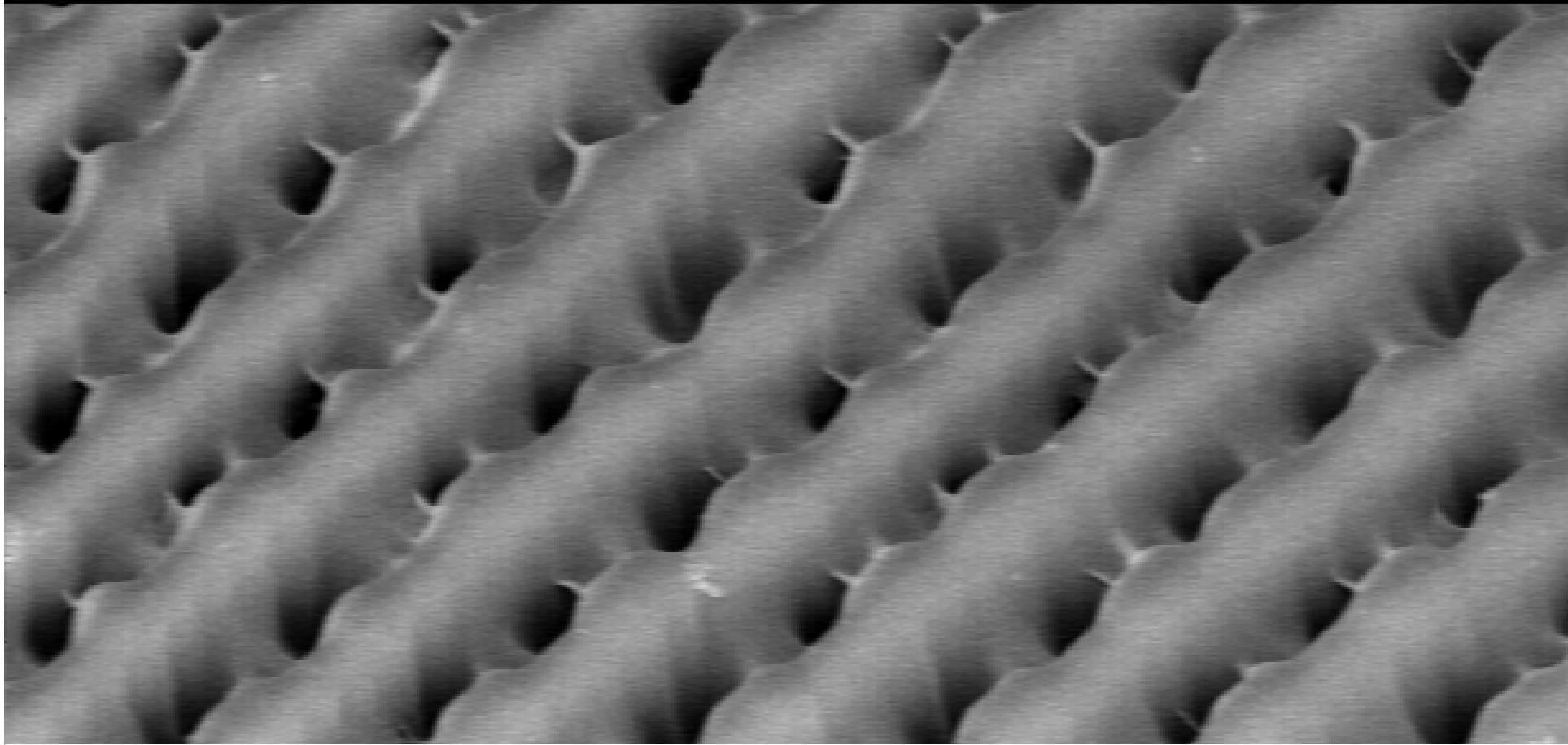
Applications of nonlinear optics

Prof. Cleber R. Mendonca



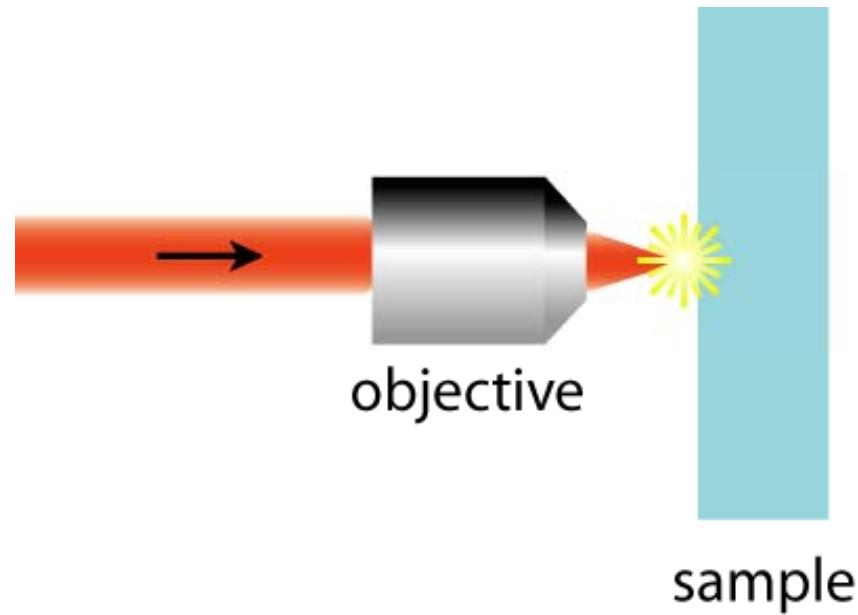
<http://www.fotonica.ifsc.usp.br>

Sculpturing with light: micro/nanofabrication using ultrashort pulses

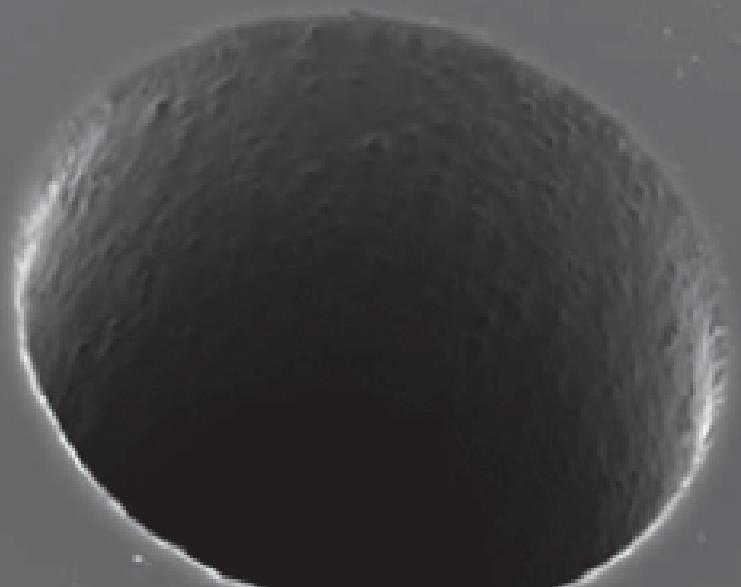


laser microfabrication

focus laser beam on material's surface

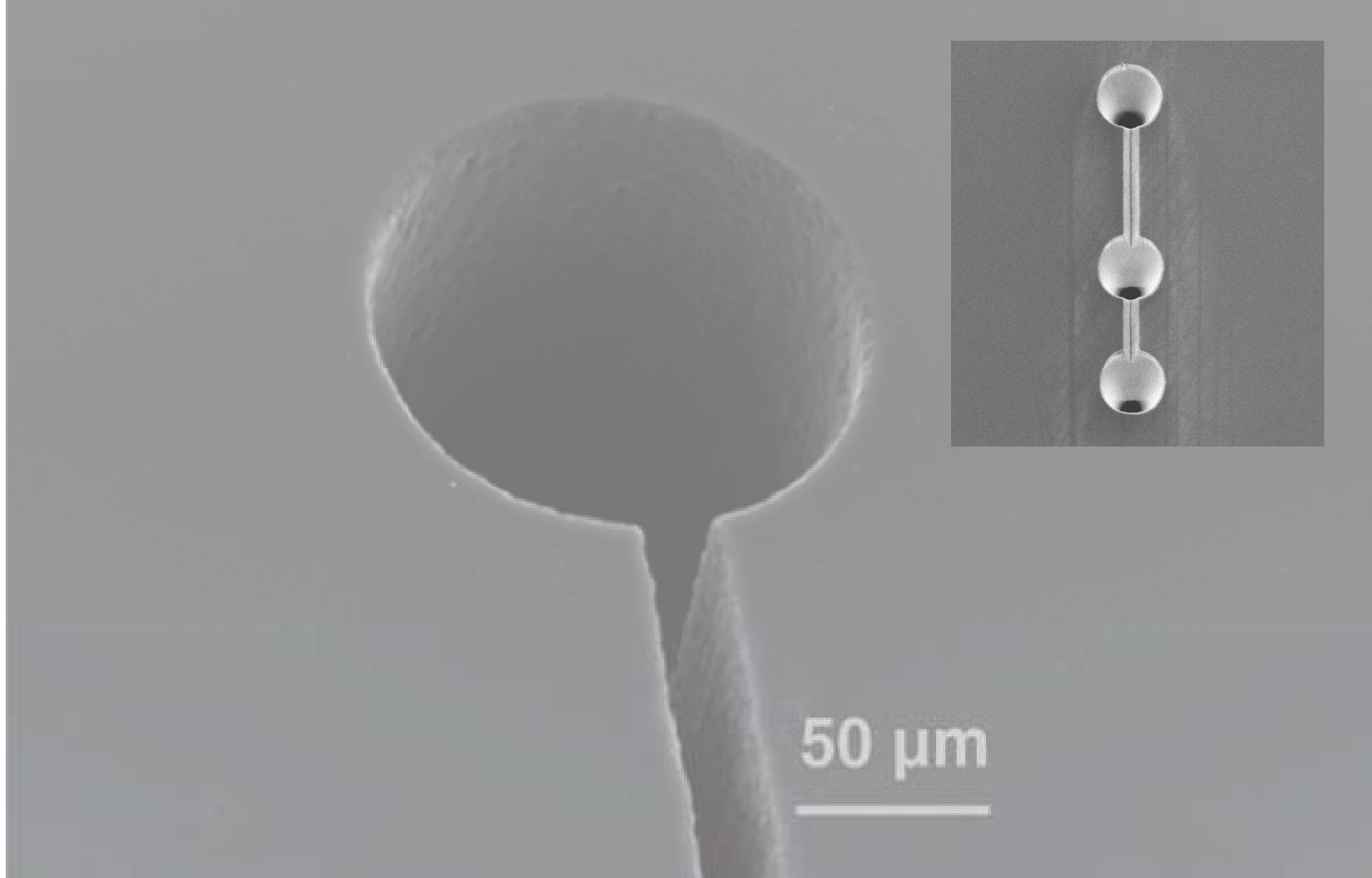


laser microfabrication

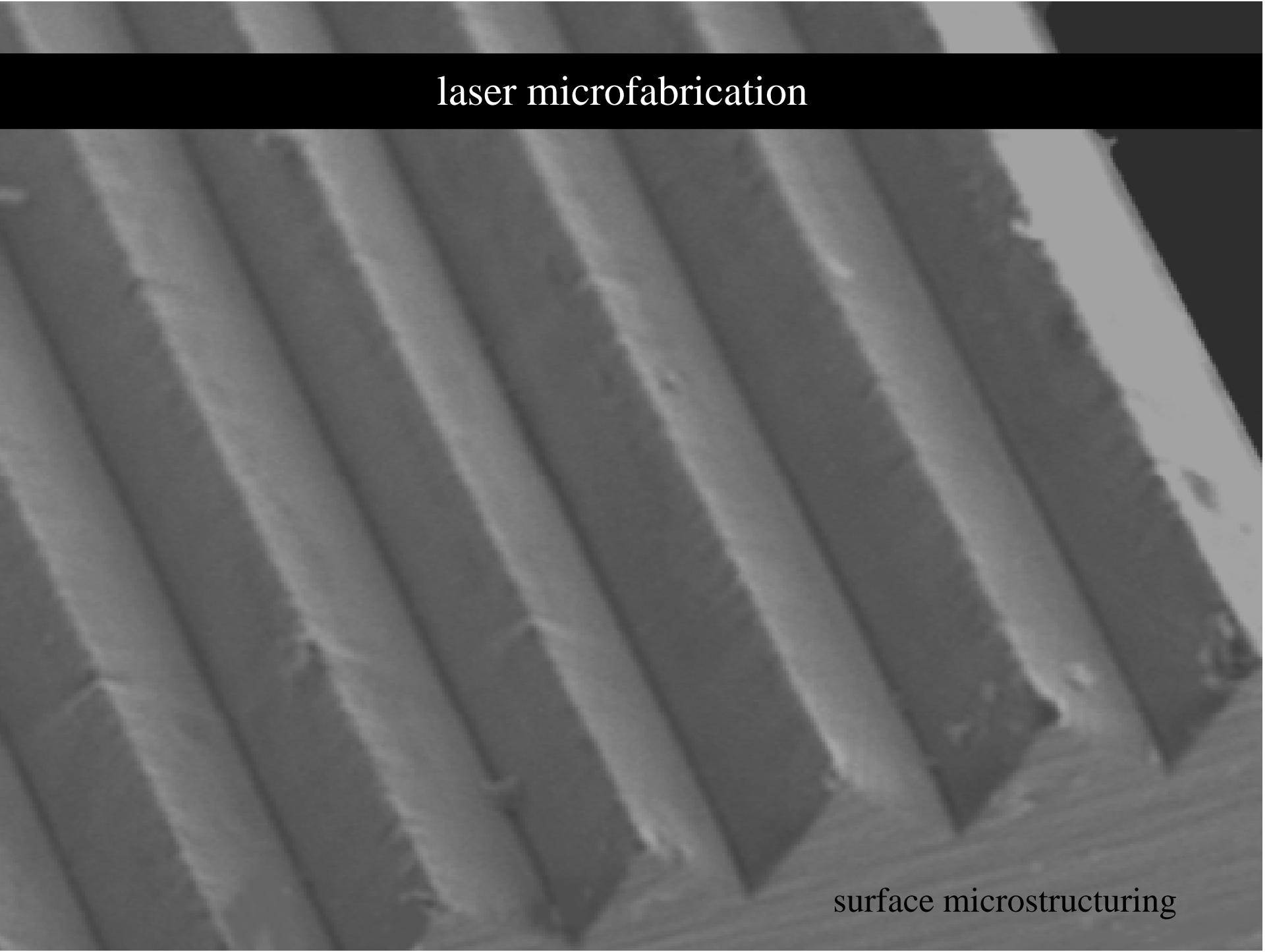


50 μm

laser microfabrication



50 μm

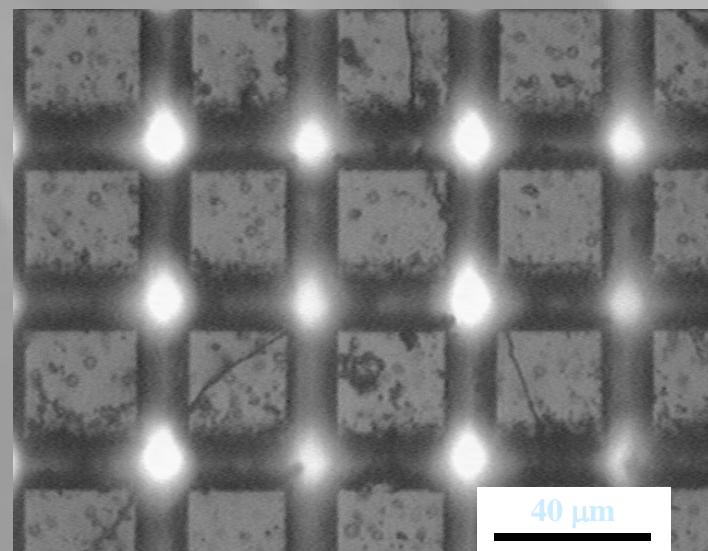
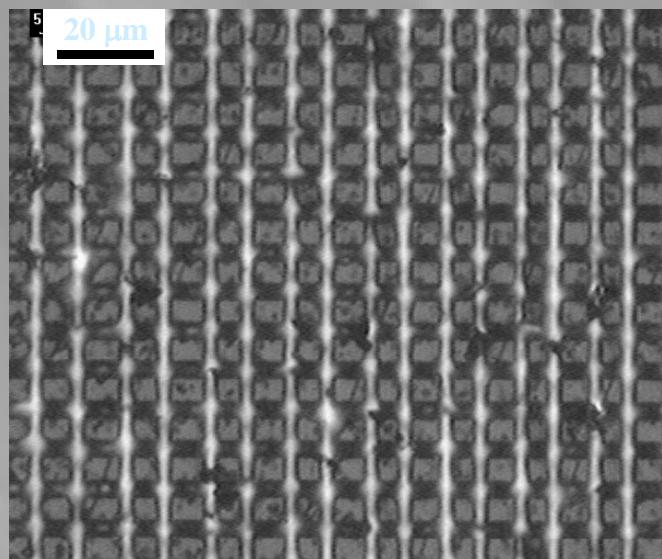


laser microfabrication

surface microstructuring

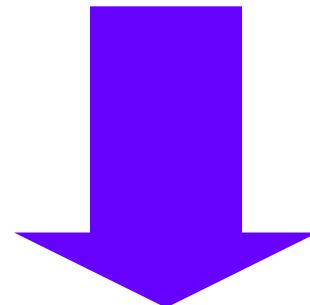
laser microfabrication

examples of fabricated surfaces



fs-laser microfabrication

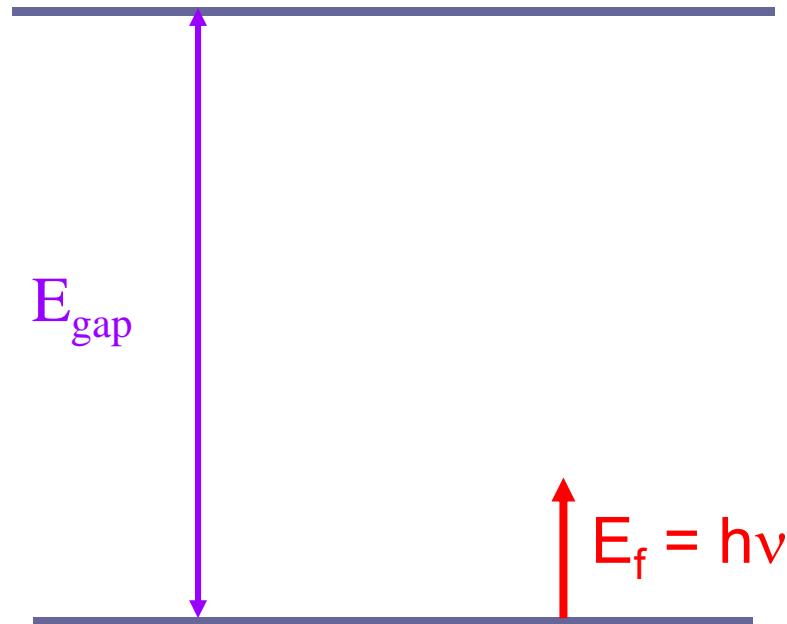
photon energy < bandgap



nonlinear interaction

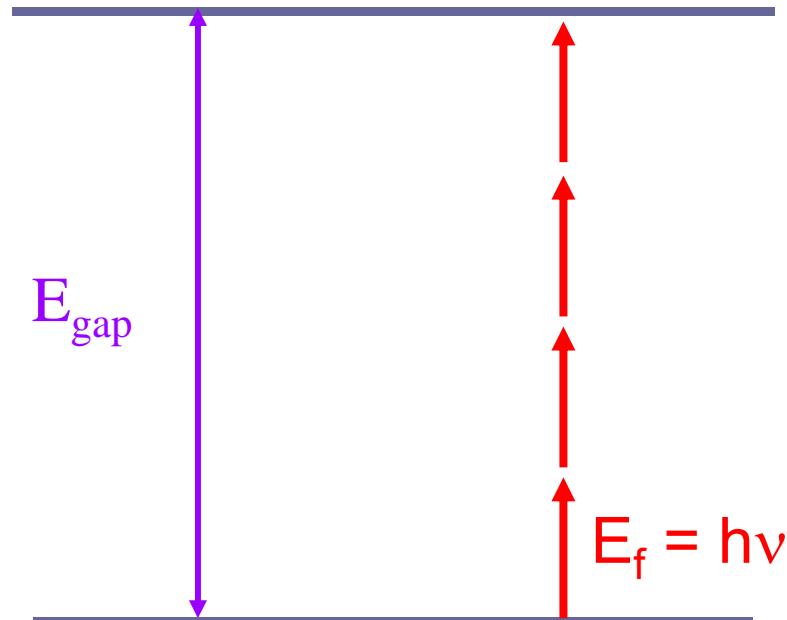
fs-laser microfabrication

nonlinear interaction



fs-laser microfabrication

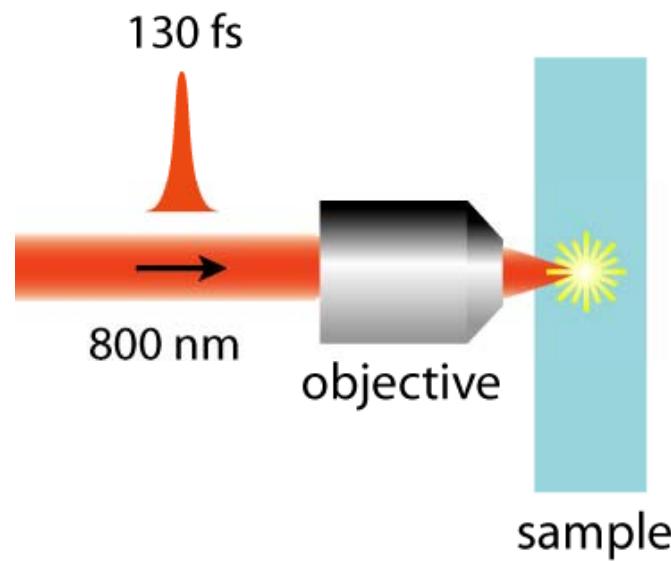
nonlinear interaction



multiphoton absorption

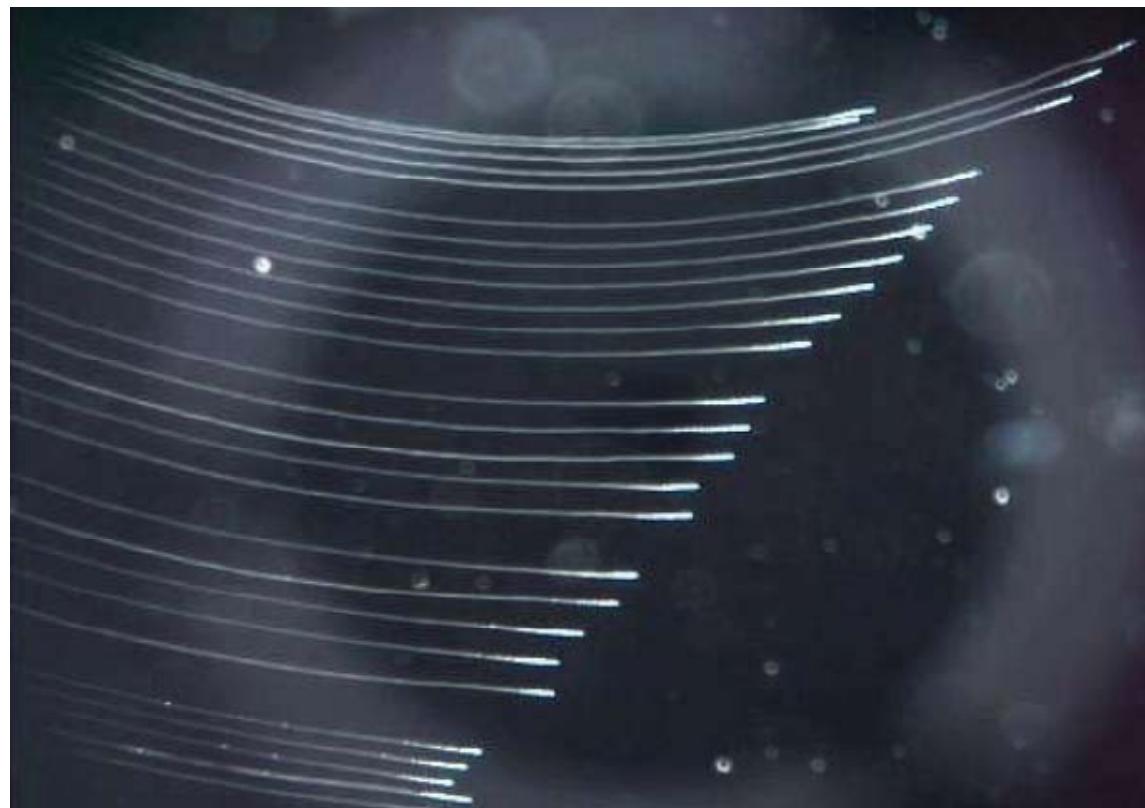
fs-laser microfabrication

focus laser beam inside material



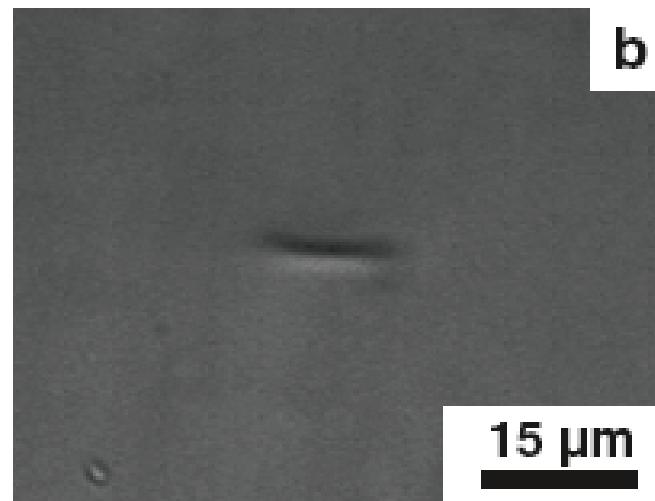
fs-laser microfabrication

curved waveguides inside glass



fs-laser microfabrication

3D waveguides in PMMA



cross-section view

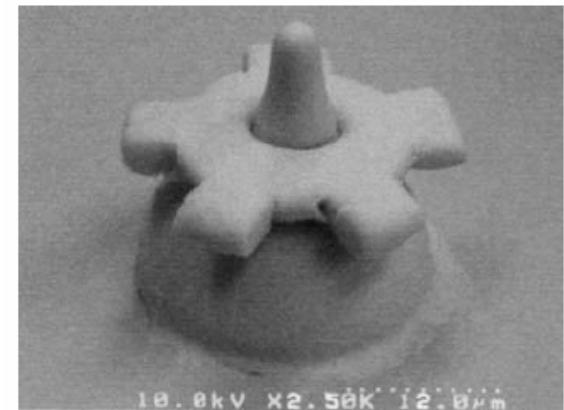
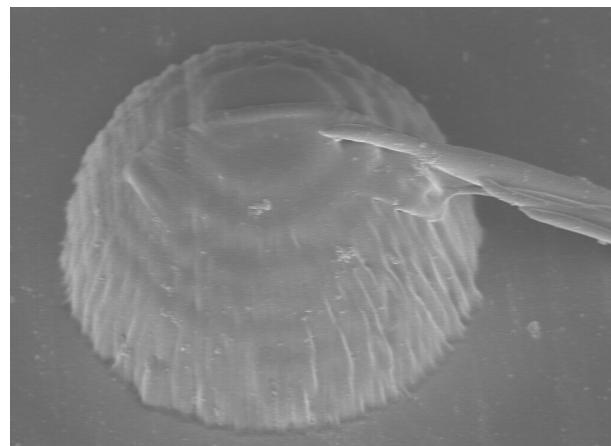
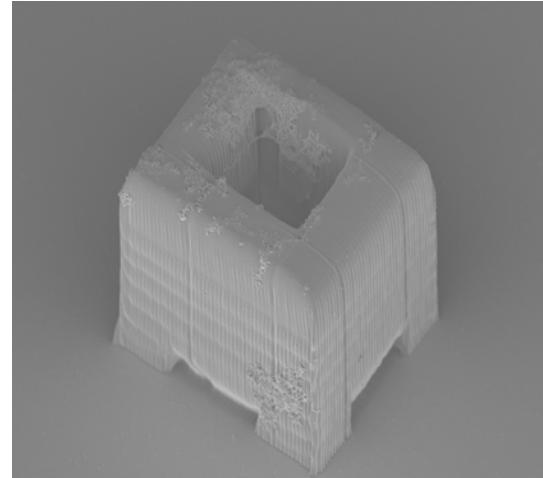
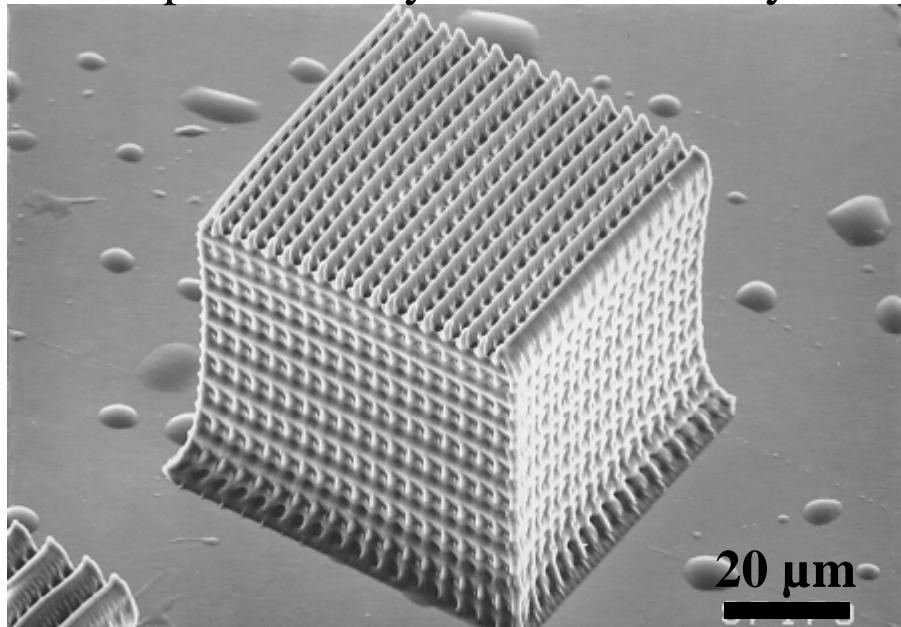
fs-laser microfabrication

Novel concept:

build a microstructure using fs-laser and nonlinear optical processes

two-photon polymerization

photonic crystal – J. W. Perry



two-photon polymerization

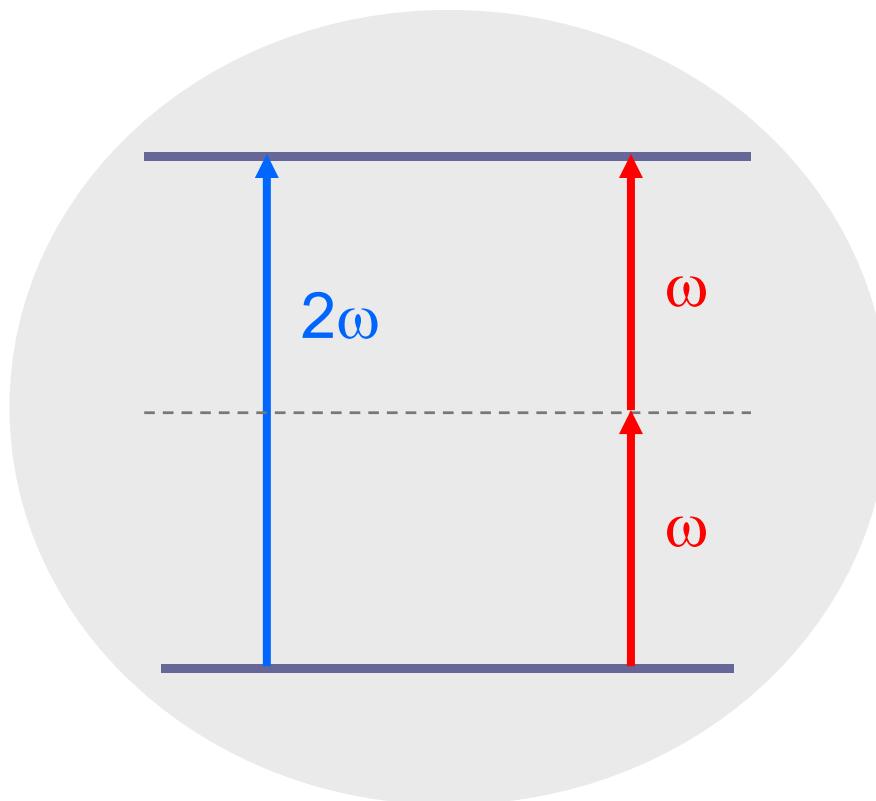
applications

- micromechanics
- waveguides
- microfluidics
- biology
- optical devices

Outline

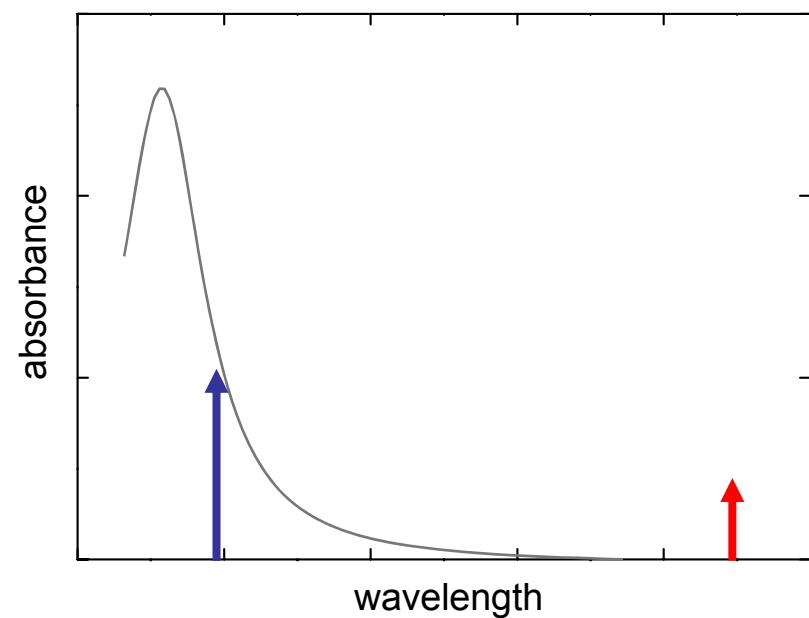
- two-photon polymerization microfabrication
- microstructures containing MEH-PPV
- waveguiding the MEH-PPV emission
- other studies
- summary

Two-photon absorption



$$\alpha = \alpha_0 + \beta I$$

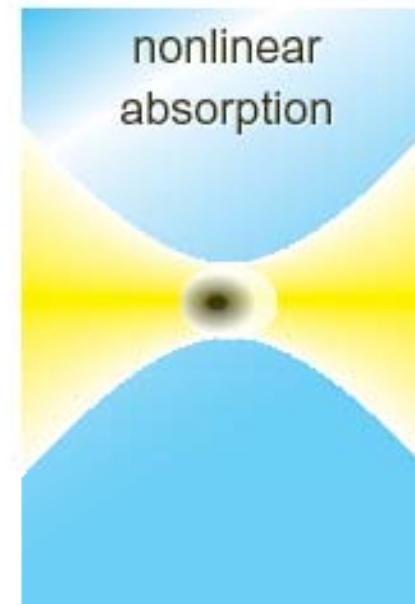
Third order processes $\chi^{(3)}$



Two-photon absorption

Nonlinear interaction provides spatial confinement of the excitation

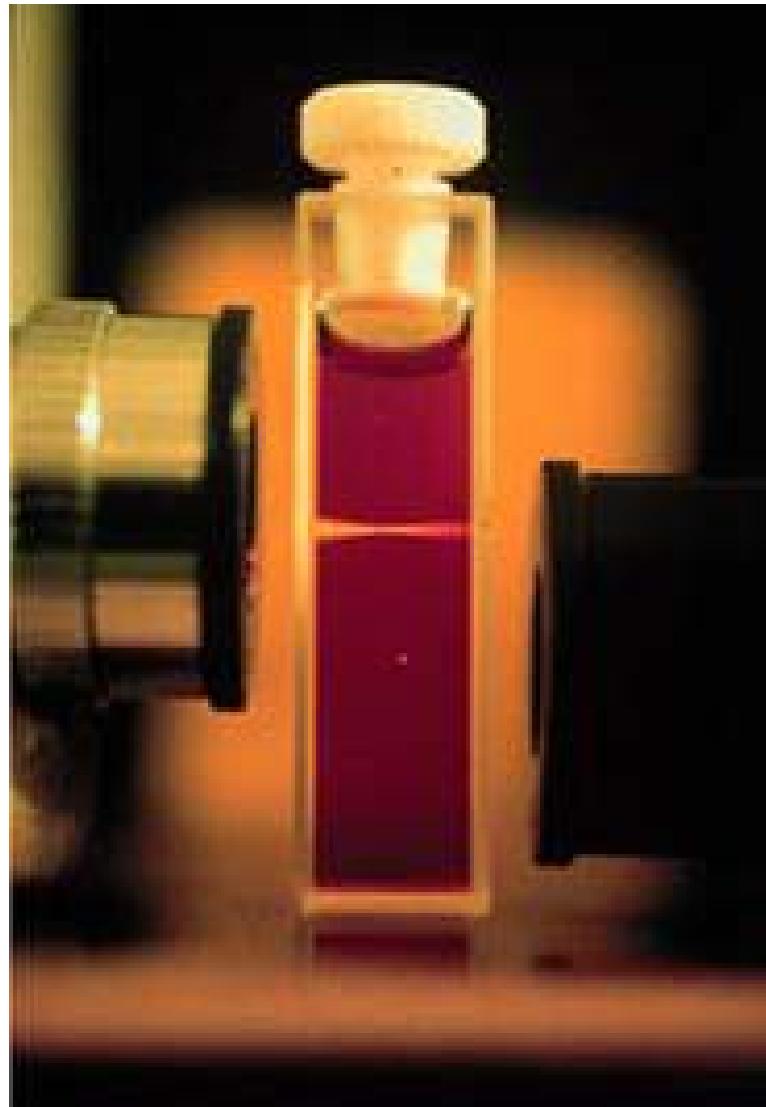
fs-microfabrication



$$\alpha = \alpha_0$$

$$\alpha = \alpha_0 + \beta I$$

Two-photon absorption



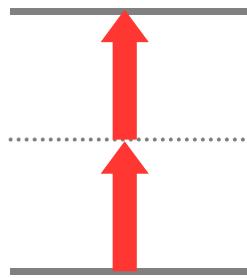
spatial confinement of excitation

Two-photon polymerization



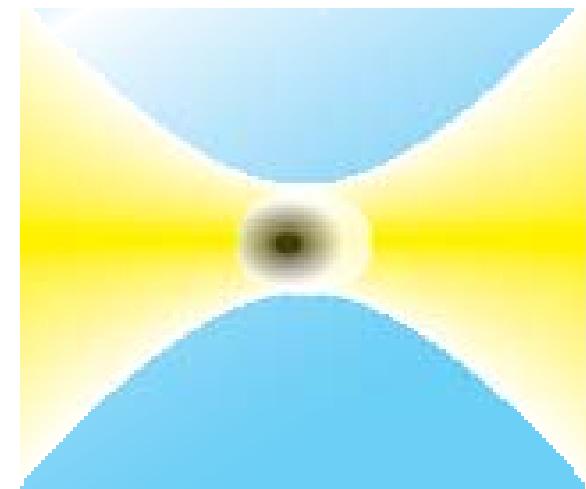
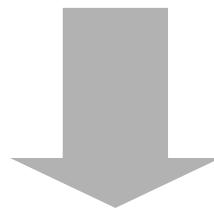
light

Photoinitiator is excited by **two-photon absorption**



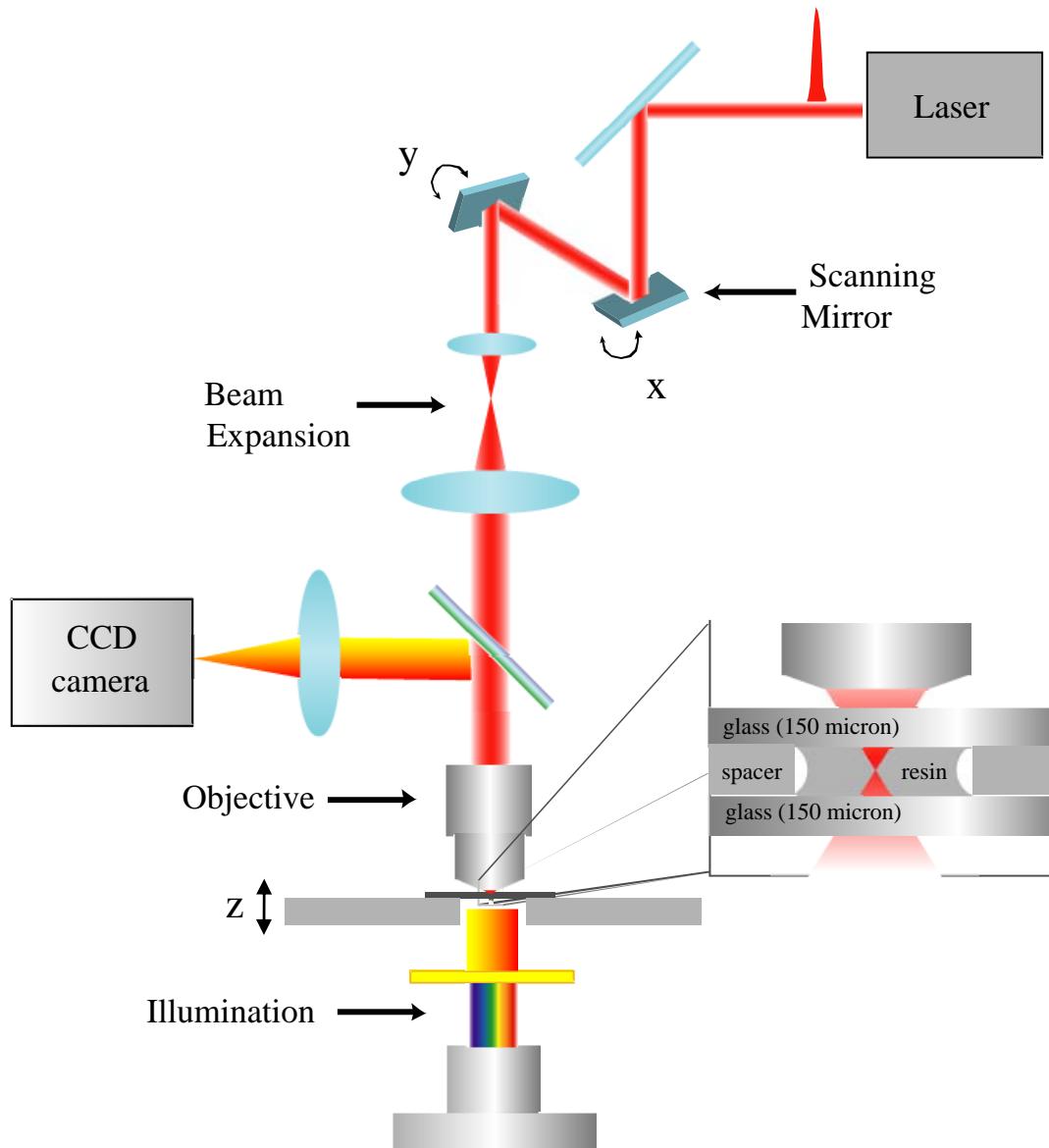
$$R_{2PA} \propto I^2$$

The polymerization is confined to the focal volume.



High spatial resolution

Two-photon polymerization setup



Ti:sapphire laser oscillator

- 130 fs
- 800 nm
- 76 MHz
- 20 mW

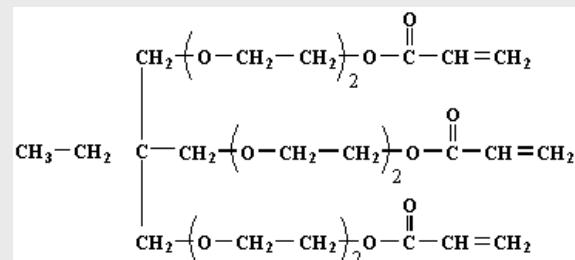
Objective

40 x
0.65 NA

Resin preparation

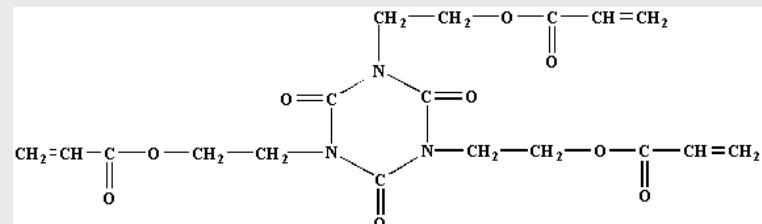
Monomers

Monomer A



reduces the shrinkage upon polymerization

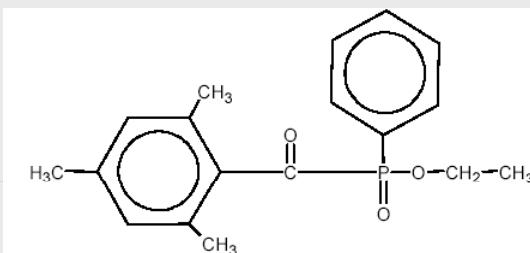
Monomer B



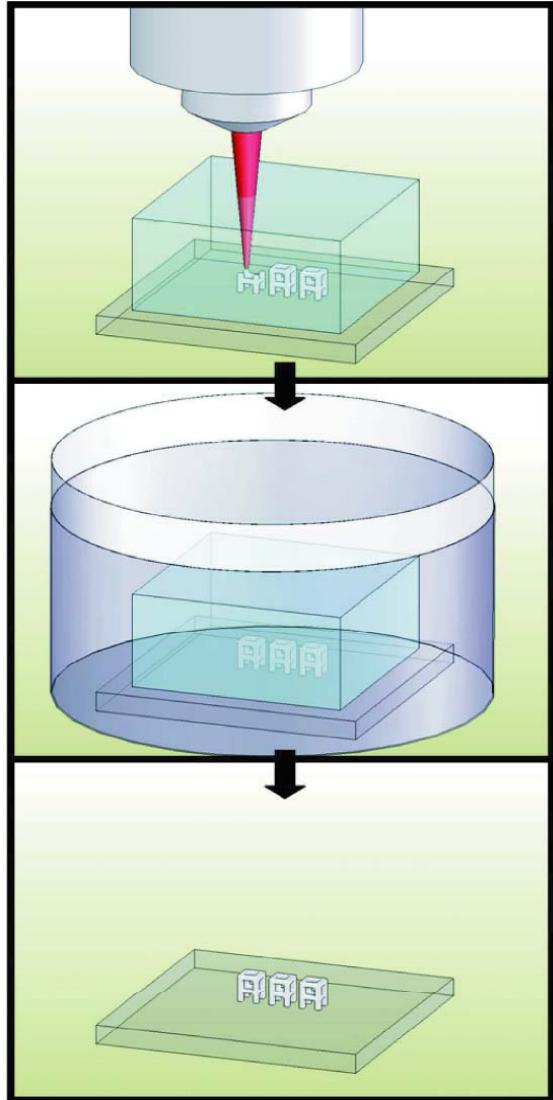
gives hardness to the polymeric structure

Photoinitiator

Lucirin TPO-L



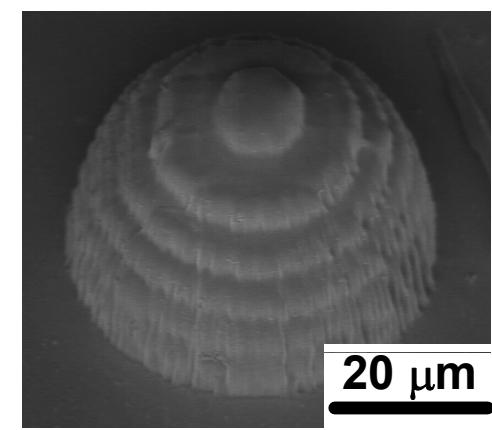
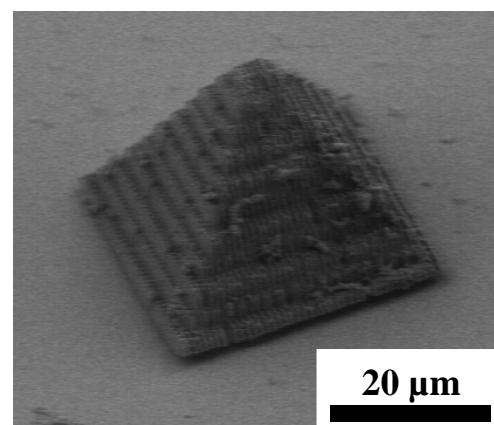
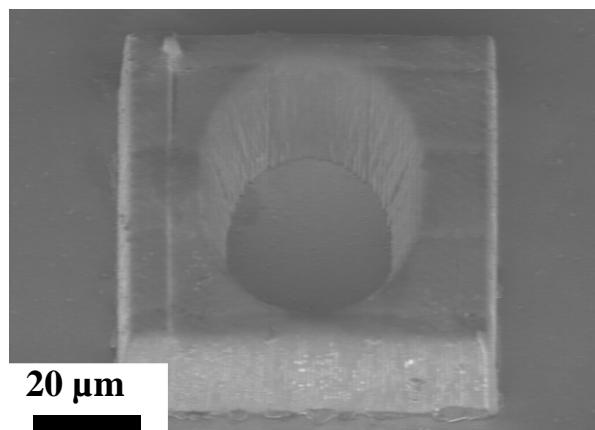
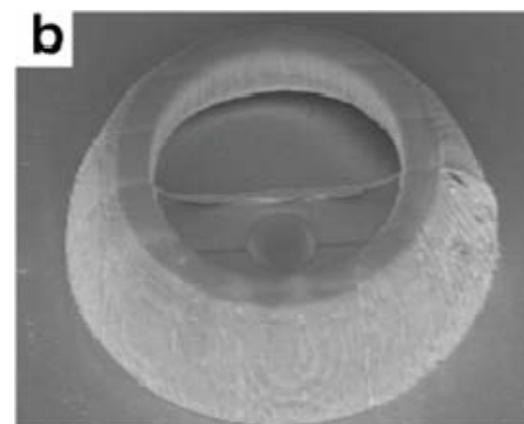
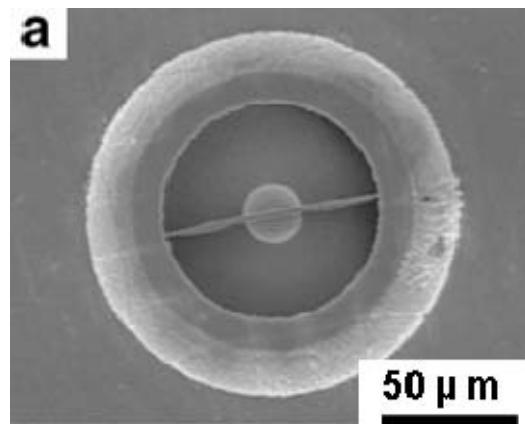
Two-photon polymerization



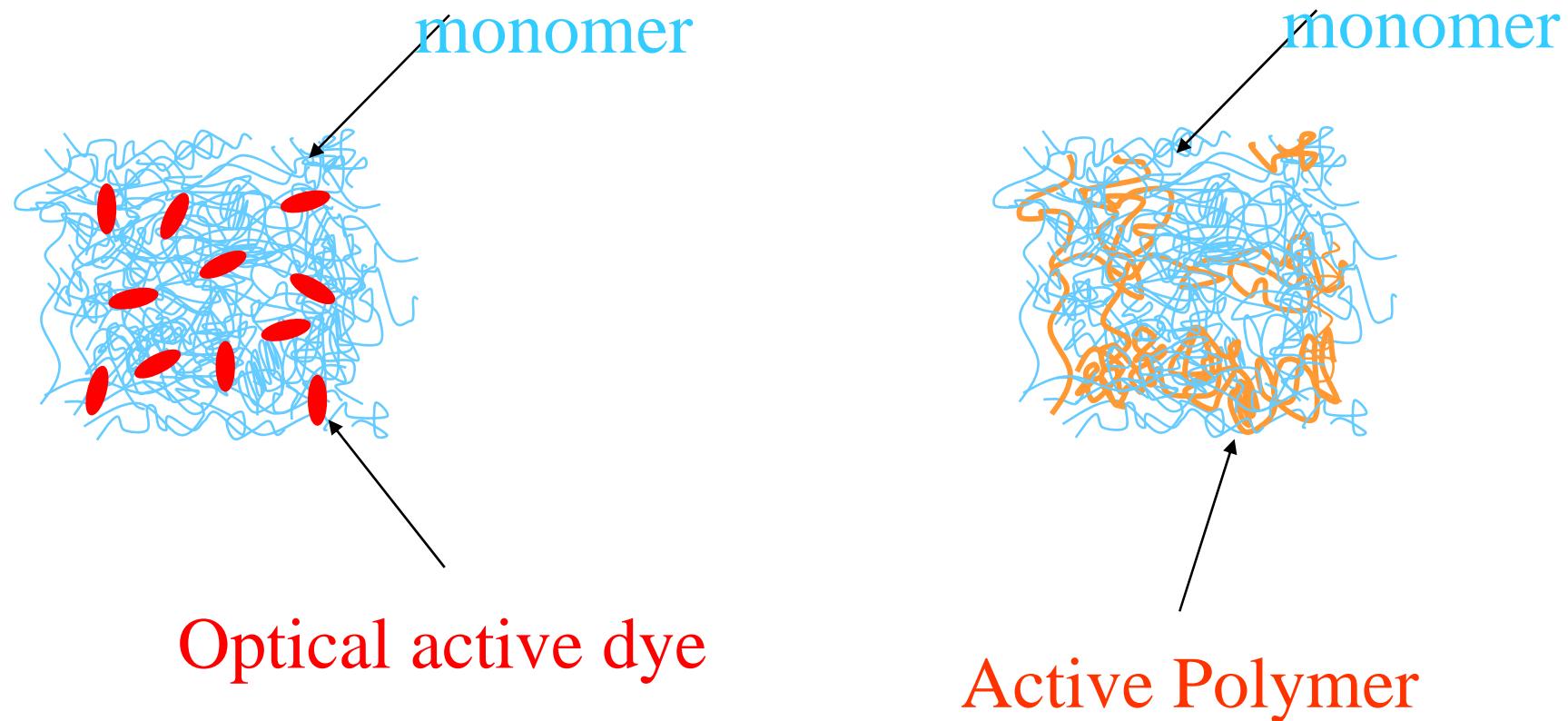
After the fabrication, the sample is immersed in ethanol to wash away any unsolidified resin and then dried

Two-photon polymerization

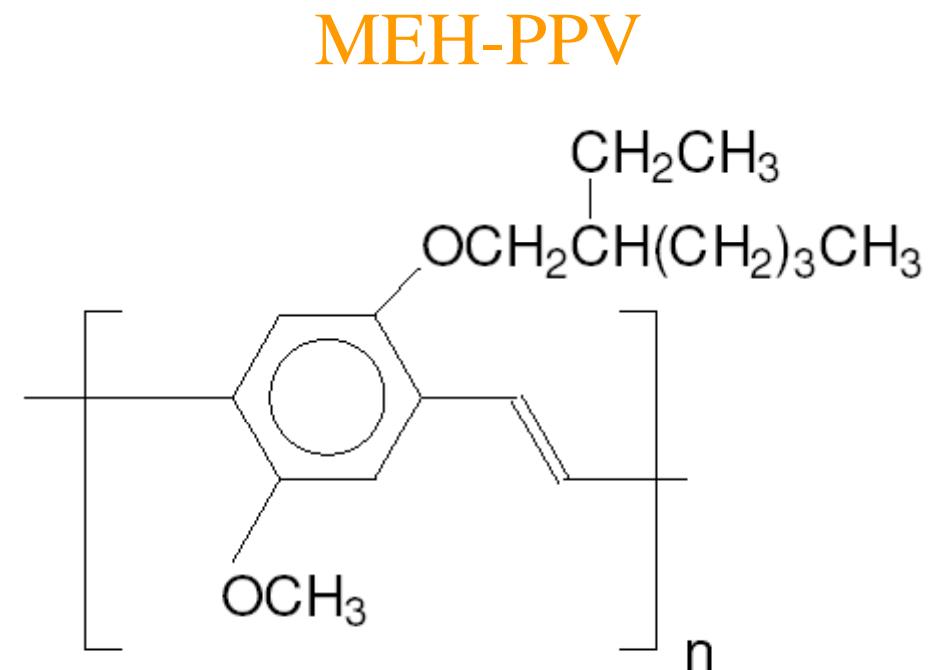
Microstructures fabricated by two-photon polymerization



Microstructures containing active compounds



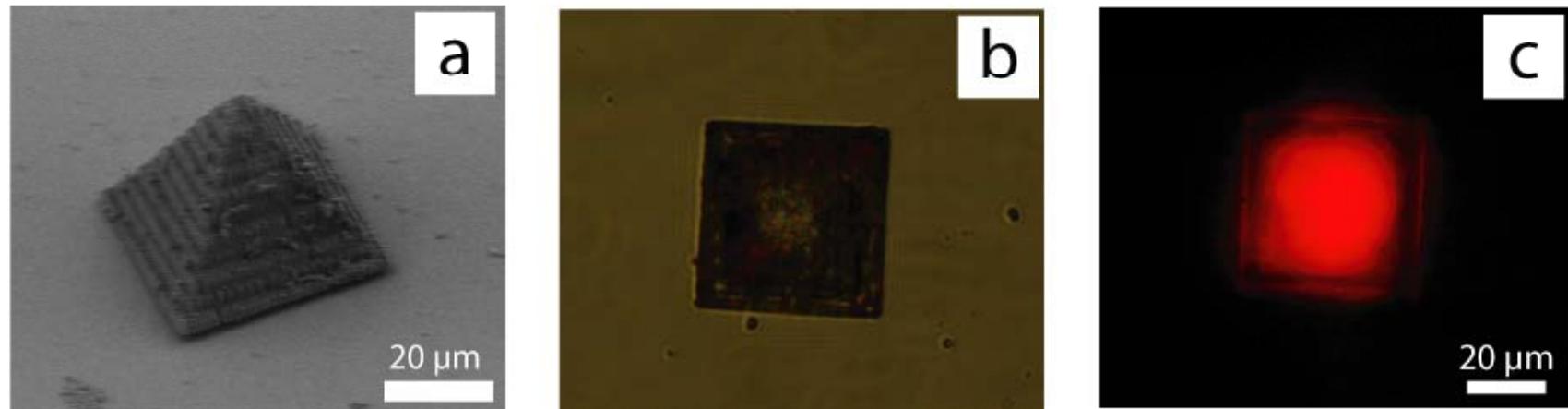
Microstructures containing MEH-PPV



Fluorescence
Electro
Luminescent
Conductive

Microstructures containing MEH-PPV

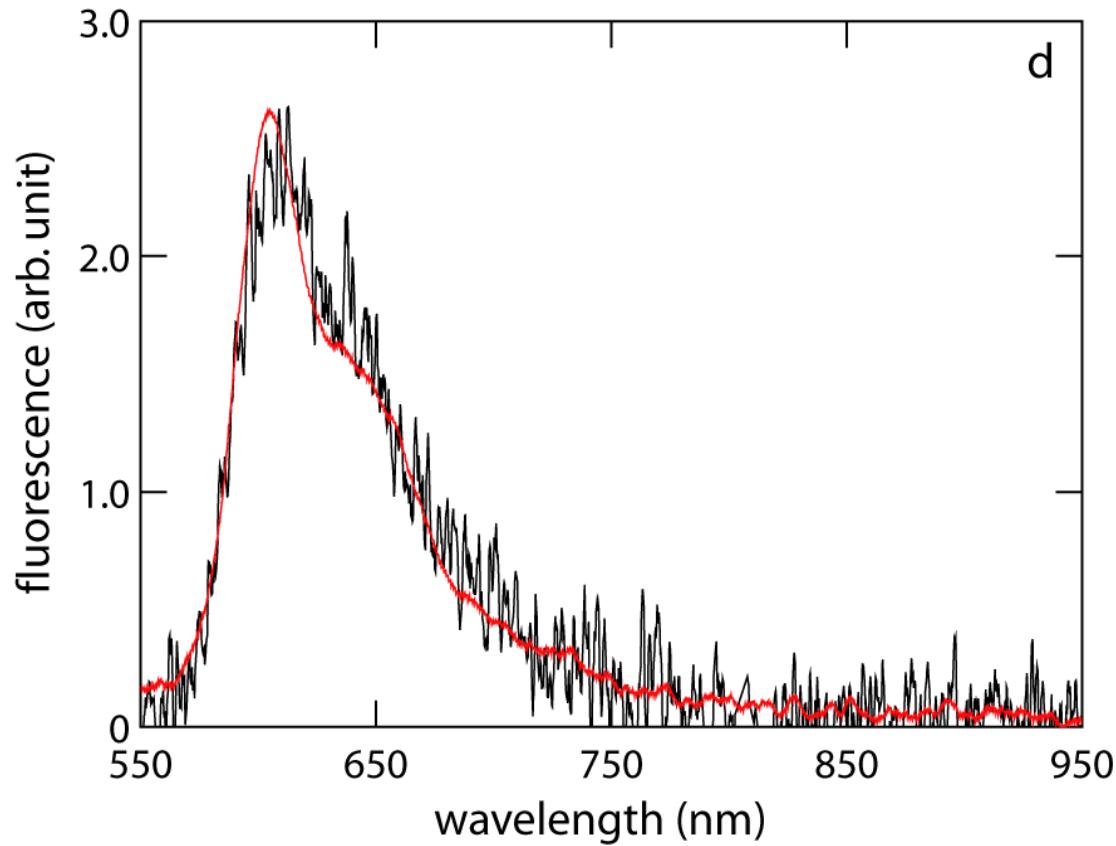
MEH-PPV: up to 1% by weight
laser power 40 mW



a - Scanning electron microscopy

b,c - Fluorescence microscopy of the microstructure with the excitation OFF (b) and ON (c)

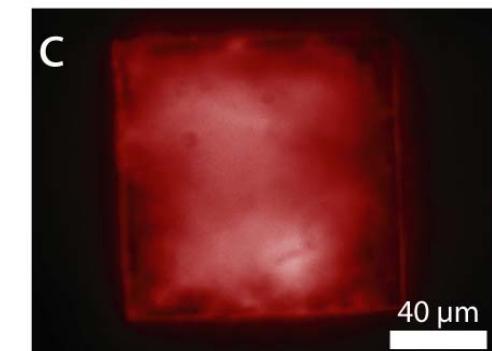
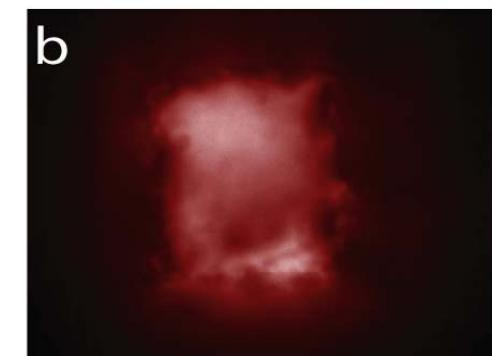
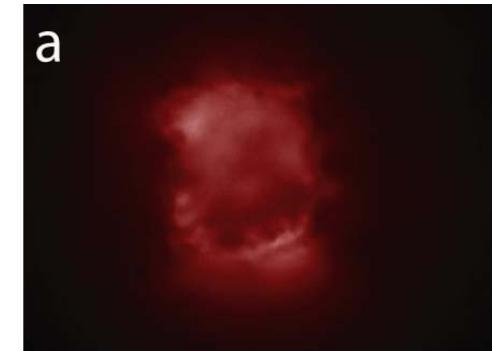
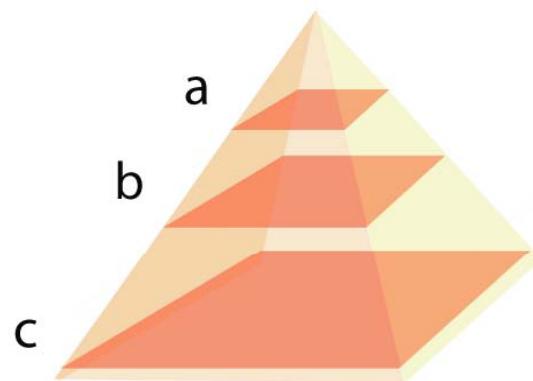
Microstructures containing MEH-PPV



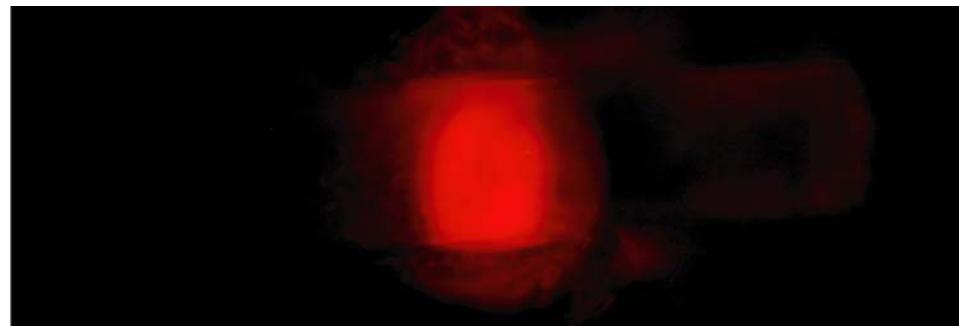
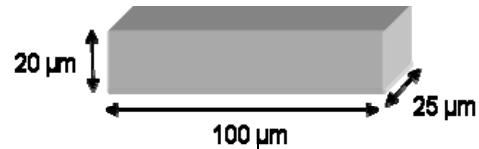
d - Emission of the microstructure (black line) and of a film with the same composition (red line)

Microstructures containing MEH-PPV

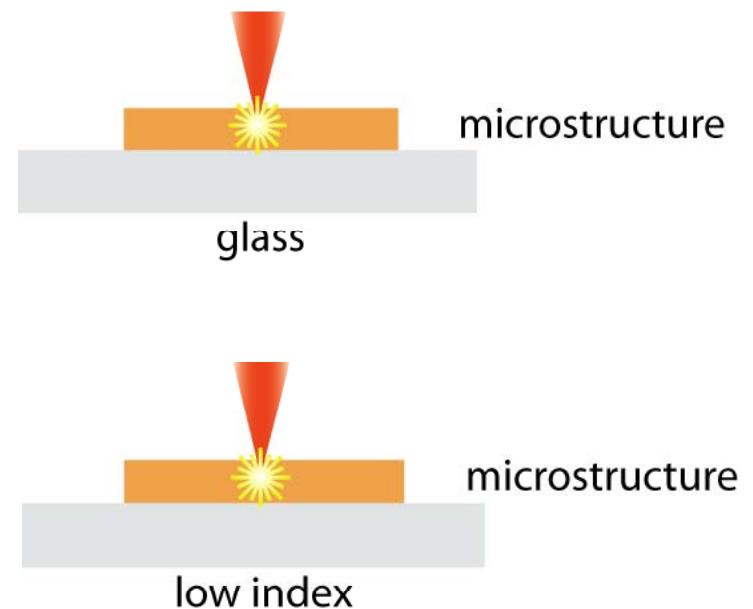
Fluorescent confocal microscopy images in planes separated by 16 μm in the pyramidal microstructure.



Microstructures containing MEH-PPV



20 μm

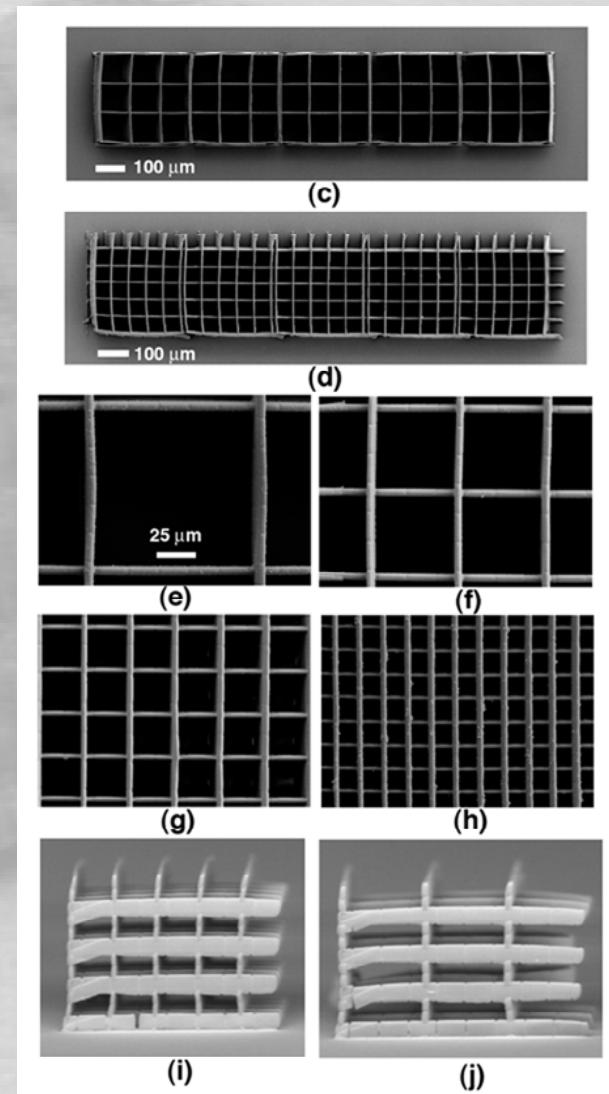
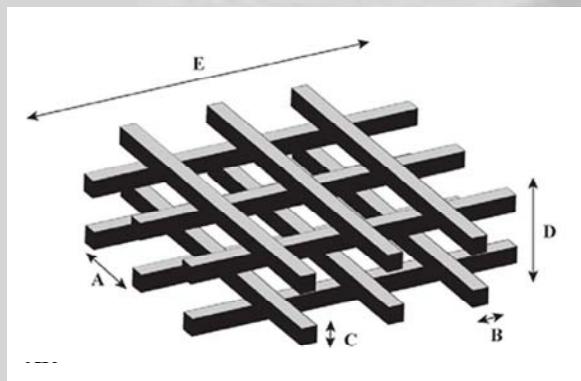


waveguiding of the microstructure fabricated on porous silica substrate ($n=1.185$)

Applications: micro-laser; fluorescent microstructures; conductive microstructures

Other studies

- 3D cell migration studies in micro-scaffolds



SEM of the scaffolds

110 µm pore size

52 µm pore size

Top view

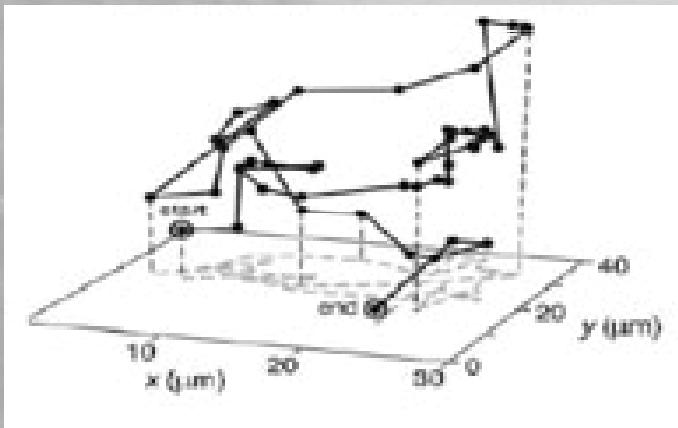
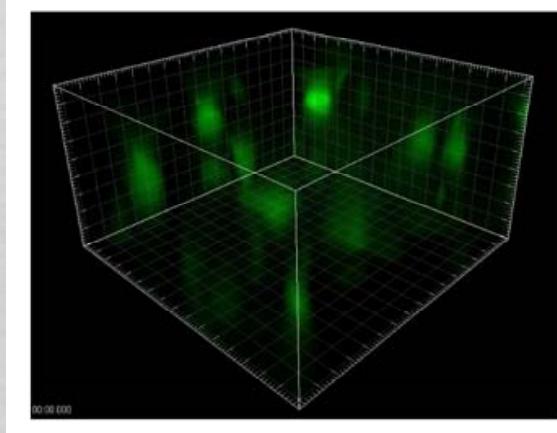
*110, 52, 25, 12 µm
pore size*

Side view

*25, 52 µm
pore size*

Other studies

- 3D cell migration studies in micro-scaffolds



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presentations

